USES OF HISTORY IN MATHEMATICS EDUCATION: DEVELOPMENT OF LEARNING STRATEGIES AND HISTORICAL AWARENESS

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The purpose of the paper is to present a theoretical framework for a systematic analysis and discussion of uses of history for teaching and learning mathematics, hereby proposing a didactical transposition of history from the academic research subject to history in mathematics education. The use of the theoretical framework is exemplified by an analysis of a project work on the history of Ancient Egyptian mathematics taught in a class of Danish upper secondary school students (10th grade), illustrating how uses of past mathematics can aid development of students’ learning strategies and historical awareness.

1. INTRODUCTION

The purpose of the present paper is to develop a theoretical framework for a systematic analysis and discussion of uses of history for teaching and learning of mathematics with respect to how history benefits students’ learning of mathematics, and develops students’ historical awareness. Several recent papers have discussed whether these two aims pose a dilemma between genuine history and relevant mathematics for teachers who want to use or integrate history in their classrooms (Freid, 2001; Jankvist and Kjeldsen, forthcoming; Kjeldsen, forthcoming; Kjeldsen and Blomhøj, forthcoming). While these discussions have focused on transforming views of mathematics and mathematics education, their conception of history has been taken to be more or less synonymous with a traditional professional historians’ approach to history – at least in the methodological approaches and the criteria for a genuine approach to history. However, perhaps we also need to broaden our view of history as well if we want history to play a more significant role for teaching and learning mathematics. In the present paper such a broadened view of history is outlined, and its implications for history in mathematics education are discussed. The aim is to develop an adequate theoretical framework for integrating history of mathematics in mathematics education that can be used to analyze specific implementations and to provide a tool for orienting the design of future implementations of the history of mathematics in mathematics education. The main focus of the paper is theoretical, but it also contains an empirical section that illustrates the theory in a carefully designed and implemented case study.1

First of all, some historiographical reflections and a position are presented. Secondly, uses of history are discussed to present a framework in which their uses for the teaching and learning of mathematics can be systematically analyzed with respect to purposes and didactical values. This discussion is based on the Danish historian
Bernard Eric Jensen’s (2010) approach to history. Thirdly, the framework is adapted to mathematics education. Lastly, to connect the discussion with the practice of teaching, a project work on mathematics in Ancient Egypt is analyzed. The project work was designed by a mathematics teacher working at a Danish Gymnasium (upper secondary level) during a professional development course in “problem based project work in, with and about mathematics”. The teacher implemented the project work in his own teaching practice in a class of first year Danish high school students (age 16-17) and documented his experimental teaching in a written report. His report will be analyzed to illustrate how uses of past mathematics can aid development of students’ learning strategies and historical awareness, thereby substantiating some of the points raised in the present paper. The paper ends with some concluding remarks.

2. HISTORIOGRAPHICAL REFLECTIONS

Mathematical knowledge is produced and used by humans; hence we can think of such activities as integrated elements of historical-social reality and of human life. We can perceive mathematical activities as creations of history as well as acts that create a history of mathematics. The development of mathematics and changes within our perceptions, views, and treatments of mathematics can to a certain extent be understood as realisations (intended as well as unintended) of goals set by people. If we want to understand historical-social processes in the development of mathematics as products of human activities, we must pay attention to intentions and thoughts of the actors, as well as their understanding of the subject matter and the context in which they performed and made their choices.

At a first sight it might seem that while such an approach can be used to study the history of sociological aspects of mathematics, such as the development of its profession in different countries and/or places or the history of mathematical journals, it cannot be used to study the history of the subject-matter of mathematics due to the universal character of mathematics. But if the development of mathematics is studied from its practice, where the historian focuses on concrete practices of mathematics, acknowledging that, despite its universal character, mathematical knowledge is produced by mathematicians, who live, interact and communicate in concrete social settings, the history of mathematical ideas, concepts and theories can also be pursued within such a framework.

Such a position is in accordance with recent trends in the history of mathematics that have emerged as reactions towards the well-known critic of the widely used anachronistic (whiggish) approach to history of mathematics and the methodological debate of internalism versus externalism (Epple, 2000), (Kjeldsen et al. 2004), and Science in Context, 2004, 17(1/2). Within the last decades many studies in the history of mathematics focus on the practice of mathematics within social, intellectual, and cultural contexts of mathematical activities. Here professional historians of
mathematics have a critical approach to source material they analyze in order to understand its significance in its proper historical context.

3. USES OF PAST EPISODES

It is not the main purpose of general mathematics education to educate and train professional historians of mathematics, and in most cases mathematics teachers will not be professional historians. In some countries development of students’ historical awareness is part of the curriculum, but that is not always the case, and if it is it only plays a minor part. With this in mind it seems too restrictive to require that the history of mathematics taught within mathematics education should be presented as traditional academic history. A didactical transposition is needed, just as is the case with school mathematics, which is also not identical with the discipline of (academic) mathematics. In the following, Jensen’s (2010) broader view of history will be introduced along with several pairs of concepts that can be useful for a nuanced analysis and discussion of the role of past mathematical episodes for the learning and teaching of mathematics.

Jensen (2010) sees the academic research subject history, as professional historians think and work with it, as just one of many approaches to history. According to him, history is employed every time a person or a group of people is interested in something from the past, and uses their knowledge about it for some purpose. People use history for many different purposes and in many different connections, and consequently there are major differences between a lay person’s and a professional historian’s use of history. Recent investigations (Rosenzweig and Thelen, 1998) have shown that lay persons’ and professional historians’ conceptions of history differ in various respects and on several levels. Lay-history has a reputation of being naïve viewed from the academic discipline of history, while on the other hand lay historians view academic history as lifeless and remote from the real world. For professional historians it is important to place past episodes and artefacts in their historical contexts. Their historical awareness is conceived of as an interpretation of the past whereas lay persons view history more as a source of memoirs.

Jensen distinguishes between pragmatic and scholarly approaches to history. In a pragmatic approach the study of the past is guided by the idea that we can learn from history. The “usefulness” of history is an underlying perspective or principle in a pragmatic approach to history. The idea is that through history we can gain knowledge about our world of today, that history can teach us better ways to live our lives. In a pragmatic approach to history, past events are studied from a utility perspective. Jensen (2010, p. 51) contrasts a pragmatic approach to history with a scholarly approach, where historians retain a critical distance to past events and emphasize differences between past and present. In the professional, academic discipline of history both traditions can be found, but since the mid 19th century the scholarly approach to history has been more and more dominant.
Observer history and actor history are another pair of concepts through which we can discuss and understand uses of past events and sources. Jensen (2010, p. 41) talks about uses of the past from an actor perspective, if we use history to orient ourselves and act in a present context. He calls this an intervening use of history. If the past is viewed retrospectively with a purpose to enlighten instead of a purpose to act or intervene he talks about uses of past from an observer perspective.2

Finally, the so-called “living history” use of history is a way of using the past to help participants develop historical awareness and learning strategies. In Denmark living history takes place at some museum centres and at some yearly events. One such centre is The Medieval Centre. On their homepage (http://www.middelaldercentret.dk/Engelsk/welcome.html) they state that the centre: “is an experimental museum where you can experience life in a reconstructed late 14th century market town: Daily life, knights tournaments, trebuchets, canons, ships, markets, … and a lot more...”. According to Jensen (2010, p. 145) living history appeals to so many not only because the participants actively take part in the events, but also because they use other types of learning strategies where the focus can be, for example, to develop the skills of past craftsmen.

4. WHAT IS THE CONNECTION TO MATHEMATICS EDUCATION?

These concepts of, approaches to, and thinking about history and uses of past episodes and artefacts present a framework for a refined discussion and systematic analysis of how past episodes and sources can be/are used in the integration of history for the teaching and learning of mathematics. They open up a variety of approaches to history and uses of the past for teachers who want history to play a role for teaching and learning mathematics. Which approach to choose depends on the intended learning. For example, Kjeldsen and Blomhøj (forthcoming) argue, based on Sfard’s (2008) theory of thinking as communicating, that history presents itself as the obvious tool for developing students’ proper meta-discursive rules, because meta-discursive rules are contingent and as such can be studied at the object level of history discourse. This presupposes a scholarly approach to history. The idea is to use past mathematical activities and sources with the intention of creating learning and teaching situations where students can experience what Sfard calls commognitive conflicts. Hence, the past is used with the purpose of intervening, and therefore the scholarly approach to history is from an actor perspective.

Kjeldsen (forthcoming) discusses the role of history for the teaching and learning of mathematics with reference to a competence based understanding of mathematics education (Niss, 2004). Here the development of students’ mathematical competence is the main purpose of mathematics education along with the development of some second order competencies, including historical overview and awareness. For the development of historical overview and awareness, a scholarly approach from an
observer perspective can be chosen. For development of specific mathematical competencies, a pragmatic approach from an actor perspective might be considered.

5. AN IN-SERVICE COURSE ON PROJECT WORK

The focus of the paper is on theoretical issues, but to illustrate the theory, a project work that was developed and implemented during an in-service course for upper secondary teachers in Denmark will be analysed. In this discussion the “living history” approach will be examined to see how it might be adapted as a way for mathematics teachers to use past episodes and sources to develop students’ learning strategies and historical awareness.

The theme for the project work was Egyptian mathematics. It was developed and tested in a classroom of students (10th graders) in the Danish upper secondary school in 2004 as part of an in-service course for mathematics. The in-service course was developed in response to a reform that was to be implemented in 2005.

Compared with more traditional ways of teaching mathematics the reform challenged the teachers in several ways: (1) Many were not used to teach either the history of mathematics or mathematical modelling, both of which having more prominent positions in the new curriculum than they had in the former curriculum; (2) they were required to bring mathematics into play in interdisciplinary projects in cooperation with other subjects, from science, from the humanities, and from the social sciences; and (3) they had to design, organise and carry out project work in their mathematics teaching. The goal was to create an in-service course where theories in didactics and pedagogy interacted with development of the participants’ own teaching practice in ways that also related to inquiry-based teaching and learning. On this basis the objective of the in-service course was to support teachers in their development as teachers, implementation in their own classes, evaluation of the project work, and documentation through a written report of a project-based and problem-oriented course in the history of mathematics or in mathematical modelling. The core element of the in-service course was the development of the teachers’ experimental practice with history of mathematics or mathematical modelling and problem-oriented project work.

The in-service course began with a three day seminar where the teachers were introduced to the history of mathematics, mathematical modelling, didactical theories, and problem oriented project work. The teachers worked in small groups developing a project-organised course in either history of mathematics or mathematical modelling of their own choice consisting of approximately 10 lessons of 45 minutes each. They decided on (1) the objectives for their own professional development, (2) their objectives for students’ learning, (3) how to “set the scene” for their own students’ project work, and (4) how to evaluate the students’ learning.

A few weeks after the seminar a first draft of the design for the project work and the materials that should be given to the students were distributed to all participants in the
in-service course. All teachers tried out their project work in their classroom. During that period there was a one day seminar to support the teachers in the documentation of their results and reflections on their experimental teaching. It all ended with a 2-day seminar, where the teachers’ written reports were discussed extensively. The final reports are published on the internet together with the handout materials for the students for other teachers to use (http://magenta.ruc.dk/nsm/uddannelser/gymnasielaerer/).

I will not go into further detail on how we define problem-oriented project work (interested readers are referred to Blomhøj and Kjeldsen, 2006), but only emphasize that the problem that students are going to work on should function as the “guiding star” for their work. In the ideal case every decision made in the project work should be justified by its contribution to the solution of the problem. This is crucial, since engaging in decisions provides opportunities for students to work independently, to gain control, and to direct the project. In order for this to happen, though, the teacher needs to set a scene for the project work, that is to formulate the task for the work, the conditions for the working process, the time constraints, and the requirements for the end product, for example a written report or a power point presentation fulfilling some specific requirements. In this way it is possible for the teacher to have some control while at the same time to leave room for the students to take responsibility and make decisions.

The in-service course is still offered with the modification that we focus only on mathematical modelling. Therefore we only have one history project to present, but since its function here is to serve as a concrete illustration of the theoretical framework developed above, and not as documentation from an empirical experiment it can be used to characterize the suggested methodology.

6. EGYPTIAN MATHEMATICS: A PROJECT WORK IN A 10TH GRADE

The project on Egyptian mathematics was developed and implemented in a classroom of 1. year students (10th grade, age 16) in a Danish upper secondary school in the fall term. The project work was meant to be interdisciplinary, with history about Ancient Egypt in combination with their mathematics. The mathematics teacher had no experience with project-organised teaching in mathematics, which was his focus for his own professional development. His objectives for the students’ learning were to:

a) enhance the students’ competence to work in teams
b) enhance the students’ independent learning
c) enhance the students’ oral presentation skills
d) have the students gain experiences with power point
e) have the students appreciate that mathematics has been different from what it is today
f) develop the students’ awareness that mathematical results have evolved, that mathematics is not static, which is contrary to the way it is often presented.

g) develop the students’ awareness that mathematics develops in an interplay with culture and society. (Wulff, 2004, p. 2-3; my translation)

The objectives fall into two parts that cover all three of the above listed challenges of the reform: the first four address competence in independent study, the development for which problem-oriented project work is an excellent pedagogical tool, whereas the last three concern the history of mathematics requirements of the new mathematics curriculum. Note that a)-c) and e)-g) are elaborated versions of some of the ICMI Study *whys*, see Fauvel and van Maanen (2000, pp. 205, 207, 211-212).

The teacher orchestrated the students’ project work in three stages:

1. The first stage was an introduction to Egyptian mathematics using a text from the students’ textbook (Carstensen and Frandsen, 2002), where the teacher introduced the Egyptians’ method of multiplication by repeated doubling, their number symbols, and their way of formulating problems (two lessons).

2. The introduction was followed by eight lessons during which the students worked in teams of four, guided by a description of:
   i) the problem formulation, which was given by the teacher (see below);
   ii) the learning objectives;
   iii) the product;
   iv) the topics for the teams.

   The teams worked independently. The teacher took the role of a consultant who could be called in for advice. When that happened he focused on posing questions and challenging the teams instead of providing answers. The problem formulation for all teams was: How and why did the Egyptians calculate? Each team worked with a chapter from a textbook on Egyptian mathematics (Frandsen, 1996), seven chapters all together treating their numerals, their methods for arithmetical operations, the 2/n-table, bread and bier (Pesu) exercises, equations and geometry. To have a whole textbook on an episode from the history of mathematics in Danish is a rare circumstance, and one of the reasons why Egypt was chosen for this project work.

3. Each team had to present its results for the rest of the class in an oral presentation supported by a power point presentation. This took up four lessons.

The first set of learning objectives deal with issues of enhancing students’ independent study skills. In his evaluation the teacher emphasized in particular that the students acquired the mathematical knowledge of the Egyptians by themselves (in contrast to ordinary teaching where he explained everything), that they “cracked the code” themselves, and that they were conscious about it. Regarding item e) and f) of the second set of learning objectives, the teacher wrote: “they were all about gaining insights into current mathematics precisely by studying the mathematics of another time” (Wulff, 2004, p. 3), from which we can infer that the teacher used a pragmatic
approach to history. He used past episodes of mathematics from a utility perspective. This also becomes clear from his description of a discussion that took place between him and the students during the introduction: “Already during the first module [the first two lessons] came the classical question, why are we going to learn this? And we had a good talk about the intended learning issues e), f), and g), during which the class apparently accepted that historical mathematics, besides being interesting as such, could contribute to a more nuanced view on current mathematics.” (Wulff, 2004, p. 5). Regarding the learning objective of realizing that mathematics has evolved over time, the teacher was rather critical, explaining that this aspect was not really complied with, since a comparison of Egyptian and modern mathematics only shows that mathematics has changed; it does not give insights into the actual process of change. Regarding the last item g) of the second part of the learning objectives, the teacher wrote in his evaluation: “here is where the subject of history can be involved. From a general knowledge about Ancient Egypt and its society, students can discuss how society and culture have been driving forces for the mathematics of that time. At the same time the historians’ method of source criticism is an essential tool for interpreting ambiguous and defective papyri” (Wulff, 2004, p. 4). In contrast to items e) and f) the teacher here takes a scholarly approach to history. The teacher used the past from an observer perspective in both approaches.

The students’ work with the sources and exercises in the textbook on Egyptian mathematics to answer the “How” part of the problem formulation can be considered a “living history” approach. They put themselves in the place of Ancient Egyptians, trying to understand and learn how they calculated, how they dealt with geometry, how they proposed mathematical problems, and so forth. The teacher reported the following situation he observed in the classroom: “Many students wondered about how “stupid” the Egyptians were. Why did they only use unit fractions? Why should a number be expressed as a sum of different unit fractions? On the other hand their methods were very difficult to understand; that is rather advanced, so in that respect they weren’t stupid at all. I think that many of the students realized that current mathematics is not “just” like today, but is a result of a long development, during which many things have been simplified.” (Wulff, 2004, p. 7). This shows a development of historical awareness among the students. That the students’ learning strategies were developed through this kind of “living history” approach can be inferred from the following observation made by the teacher: “This [that mathematics had made progress] became especially obvious when the students constantly rewrote the Egyptian notation to current notation with \( x \)'s, formulas, etc. After they had finished an Egyptian calculation they would say: ‘but that just corresponds to …’ followed by a solution of an equation in our way. It was very inspiring to see how students, who normally were a bit alienated towards \( x \)'s and equations now had taken those to themselves as their own, and all of a sudden perceived equations as an easy way to solve problems. The
students became aware that modern notation makes the calculations much easier than they would have been otherwise” (Wulff, 2004, p. 7).

As mentioned above the teacher found that item g) in the list of learning objectives, which was supposed to link the development of mathematics with a scholarly approach to history, was not realized. The “why” part of the problem formulation was designed especially towards this goal. The mathematics teacher had hoped that the students would have been able to experience concrete examples of how needs of society sometimes act as driving forces for the development of mathematical ideas. This is a very ambitious goal, and since the history teacher focused more on religion and dynasties, the mathematics teacher felt that the students did not get opportunities to gain real insights into why mathematics was developed in interaction with the needs of society and culture. A less ambitious teacher would probably evaluate this part differently, pointing towards the fact that was explained above, that the students gained genuine historical knowledge about Egyptian mathematics situated in the proper historical context. Finally, the teacher concluded that the students afterwards showed signs of possessing a more mature and reflective approach to mathematics than they had before. Unfortunately, the teacher did not document this with observations from the classroom.

7. CONCLUDING REMARKS

The purpose of the paper was to present a theoretical framework for a systematic analysis of the uses of history for teaching and learning mathematics in order to propose a didactical transposition of history from the academic research subject to history in mathematics education. The analysis of the teacher’s report on the project work on Ancient Egyptian mathematics with respect to the described framework of different uses of past episodes shows that in this project, history was used in different ways to provide a very rich teaching and learning environment. The teacher used different approaches to history and used past episodes from various perspectives for different purposes, thereby creating learning situations that developed students’ historical awareness and mathematical learning strategies at the same time. History was used in ways in which students gained genuine historical insights, developed learning strategies, and enhanced their mathematical problem solving skills even though they worked on mathematics that might not be part of the core curriculum.

NOTES

1 I would like to thank Costantinos Tzanakis for helpful comments on an earlier version of this paper.

2 A fourth pair of concepts is identity neutral vs. identity concrete history writing, which will not be used in this paper.
REFERENCES


