TRANSITIONS BETWEEN MICRO-CONTEXTS OF MATHEMATICAL PRACTICES[1]
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Abstract
The study presented here is concerned with transitions between micro-contexts of mathematical practices. These micro-contexts are determined by the use of different software. Here we focus on a task in which students had to depict the blueprint plan of their schoolyard. The task completion demonstrates a productive interaction of tool use which combines instrumental approaches to achieve the given purpose. At the same time it provides a framework for the observation of knowledge and skills transfer during transitions from one micro-context to another.

INTRODUCTION
The way digital technologies impact or could possibly impact processes of mathematics learning / teaching has been one of the main interests of the educational research over the last two decades (Artigue & Bardini, 2010). The use of digital material is not just an added tool, but creates new conditions which deviate from the established teaching and learning environments. Some of the changes caused involve the cognitive processes, the mode of production and reproduction of knowledge and the communication behaviour of teachers and students. The teachers redefine their roles and the learning process by creating new communication situations. The role of the teacher as knowledge carrier changes and what comes at the heart of the whole process is the interaction between teacher and students as well as between students themselves through the use of digital tools (Milionis, 2001).

The role and use of tools in the educational process was discussed in the previous CERME WG7 as well. It was considered important to address several issues such as design, articulation between design and use, interaction between resources and teachers' professional practice, technologies, tools and students’ mathematical activity. The role of the tools and their transformation when used for the carrying out of specific activities comprise the major issues currently of concern to researchers in the area of ICT. The tools used in the present research were: a) Geometer's Sketchpad, a dynamic geometry environment and b) Microworld pro, a Logo-based environment /Turtle Geometry software. The teaching contexts in which the materials were converted from artefacts to instruments by the students were considered to be micro-contexts. We investigated how each context relates to the way students manage mathematical operations as well as whether and how mathematical knowledge is transferred from one micro-context of mathematical
practice to another. We also looked into the process of interaction between students as well as between students and teacher as mediator.

The survey was carried out outside school hours and classroom context as part of a school environmental program called "Environmental journeys through mathematics and technology" and attended by 22 students, 13 of which were 7th Grade and 11 were 8th Grade. Meetings were held in the school computer room. The Math/ICT teacher was actively involved along with a Science teacher. The meetings took place for two hours each week for an entire school year.

Utilizing ethnographic and teaching experiment techniques (Chronaki, 2008) we studied the community of practice (Wenger, 1998), formed by the participant students and educators, while we introduced activities. The present paper summarizes the overall research effort and discusses research results focusing on one of the research tasks in which students had to depict the blueprint plan of their schoolyard.

THEORETICAL FRAMEWORK

As Artigue (Artigue, 2009, p. 1171; Guin, Ruthven, & Touches, 2004) notices researchers have become more and more sensitive to instrumental approaches (Verillon and Rabardel, 1995) to mathematical education, that is the processes of instrumentalisation and instrumentation that drive the transformation of a given artefact into an instrument of mathematical work. This perspective combines both the Piagetian and the Vygotskian theoretical frameworks. The instrumental approach is based on the distinction between an artefact and an instrument. The term artefact describes a human-made object, either material or symbolic. The term instrument describes a mixed entity with artefact-type components as well as utilization schemes (Trouche, 2005), which indicate the functional value of the instrument for the individual. These schemes concern the strategies developed by the individual in order to carry out a task. Utilization schemes are formed gradually through the use of the artefact. As a result the instrument is a mental construction of the individual and has psychological qualities. The process of the transformation of an artefact into an instrument is called instrumental genesis. This approach, is made of two interrelated processes: an instrumentation process (the artefact shaping a user’s activity) and an instrumentalisation process (the artefact shaped by the users’ activity). During the instrumental genesis process the role of the teacher proves to be of paramount importance. Trouche (2004) introduced the term "instrumental orchestration" in order to describe the teacher's management of the individual instruments in the collective learning process. He defines instrumental orchestration as a deliberate and systematic process of organizing the different artefacts which are available for a specific project. There are two ways of organizing, namely didactic configuration and exploitation mode. The first relates to the selection and
arrangement of the artefacts for the project while the second refers to the decisions made by the teacher about how the artefacts can be used to achieve the learning objectives. (Drijvers, et al., 2009) introduce a third way of organizing, which they named "didactical performance, in order to stress the ad hoc decisions taken while teaching on how to actually perform the enacted teaching in the chosen didactic configuration and exploitation mode". According to this perspective, instrumental orchestration aims at enhancing the students' instrumental genesis but also causes the instrumental genesis of the teacher/researcher himself/herself both in the process of preparing a deliberate intervention and while it is actually taking place. In this paper we focus on the on-going didactical performance mode of instrumental orchestration and discuss the results with an emphasis on instrumental jumps, which describe situations where an instrument used within a context gives rise to instrumental genesis in a different context. The contexts in the specific research are defined by the two types of software mentioned earlier. We chose these types of software because they mark two different environments both in terms of user access and epistemological foundation. In Microworlds pro, in order to create events students have to describe them in a symbolic way through scripting. In contrast, in a Geometry Sketchpad environment they can operate directly using basic geometric concepts and explore objects and their relationships through dynamic manipulation. Our intention to focus on instrumental jumps stems from the question whether and how knowledge is transferred from one mathematical context to another.

**METHODOLOGY- THE SETTING OF THE RESEARCH**

As our interest lies, on the one hand, on studying learning in specific contexts and, on the other, on exploring how students involved in learning activities think, we believe that anthropological research orientation is a suitable kind of approach. The ethnographic survey was supplemented by the implementation of teaching experiment techniques and the researcher intervened at various stages using pre-designed activities which intended to trigger events to be studied. The role of the researcher was principally that of a participant observer, who focused either on observation or on participation accordingly through the assignment of relevant activities or the learning of some software. The ethnographic equipment used included video and tape recordings, field notes, and the material handed in by the students both in final form and during the tackling of an activity.

As mentioned earlier the research was incorporated in an environmental program and so it developed along two mutually assisted directions. The environmental direction sustained the learning community for a long time while the research direction seemed to be augmenting the design of teacher intervention as well as the support of pupils. The students had the opportunity to engage in meaningful instructional situations, and teachers were supported in the design of their interventions. This mode of intervention substantially reduced the researcher’s participation in defining
and developing the activities, giving him more leeway for participatory observation. What is more, the participation of students from various grades prevented teachers from applying purely formal teaching. So the community formed differed from a typical class and enabled us to systematically organize communicative relationships between pupils of the 7th and 8th grades and to observe students in a less formalistic framework.

The research consisted of three phases based on constructionism theory concerning learning through construction, 'learning-by-making' (Halel & Papert, 1991). The first phase involved designing digital games using symbolic expression software, Microworld pro. The second phase focused on the construction of paper digital cameras pinhole camera [2] and the use of dynamic geometry software for their digital modeling. In the third phase, certain tasks derived from the interweaving of research and the environmental program was given to the students so that they could use the instruments developed in the previous phases. This paper discusses the results of one of the tasks of this third phase, namely the dynamic depiction of the schoolyard. The examples (episodes) that used to illustrate the discussion are taken from the work of two groups of grade 8 students who chose to use the environment of dynamic geometry software. This project arose from the need of the environmental program to design a dynamic depiction of the school with the aim of dynamic information registration. Such information could contribute to immediate awareness of current failures and problems in the school such as burnt bulbs, consumables shortage, and full recycling bins. This plan could be accessible to all members of the school community. The potential for dynamic scale manipulation of the floor plan derived from the need to view the site both centrally and in certain parts. To this end, students were given a real architectural sketch of the floor plan of the school as shown in figure 1. The mathematical concepts that were negotiated were those of reduction, growth and measurement.

![Fig. 1 floor plan of the school](image1)

![Fig. 2 final plan of student M5](image2)
Episodes/ Discussion of the Episodes

Although the students’ dialogues are of great interest as they reflect the students’ effort to decode the blueprint given to them, in this paper due to space constraints we focus our attention on some of the episodes associated with the drawing of the plan. Specifically, students in the two groups, three boys (M1, M2, M3) and two girls (M4, M5) are working at adjacent positions and are able to communicate with each other. Before reviewing a few episodes, we present a typical episode and an initial analysis of it. It focuses on a snippet dialogue developed among three students trying to choose the starting point and design technique. The dialogue is illustrative of the different instrumental approaches that students develop through discussion.

Episode 1 and its Analysis

404 M1: Where to start?
405 M2: From this angle.
406 M2: Let’s put a point here.
407 M3: We must measure the distance, what to do with the point?
408 M2: So I’ve put the point here, how do I tell the point to move to the left, how long is this? I want the ruler.
409 M3: Have you confused the programs? The point is not a turtle, get the ruler.
410 M2: There may be a command for the point to move.
411 M3: Hey! The ruler, what do I measure?
412 M2: This is the yard. Measure, here. Come on, measure.
413 M3: 11.2 cm.
414 M2: Do we have to tell the point to move 11.2 to the left? I don’t know.
415 M3: Here we go again, after I told you the point is not a turtle.
416 M3: Draw a line (he means segment), measure it.

The students have never before used the specific software to depict blueprint plans that require precise measurements. The dialogue focuses on how the two students differently understand how to start the drawing of the plan. The M2 student wants to start his plan from point A, as shown in figure1, giving this point a command to move [2], while the student M3, as revealed at the end of the dialogue [3], wants to begin his construction with a random segment. The attitude of the student M2, who remains anchored in the culture he had acquired about the movement of the turtle according to specific commands in the first phase of the research, reveals four different aspects. The first is the symbolic representation of a point as a turtle in the
logo-based environment. For the student, this concerns not only a one-to-one point correspondence but also a transfer of the qualities of the turtle resulting from its use. The second concerns the intention of the student to bring that culture to the new situation. The third concerns the communication of his idea to the other students in an effort to elicit ways to adjust the use of the turtle. The fourth aspect relates to the fact that his idea acquires communication features through interaction. A first example of this is the discouraging response of student M3. So starting with a problematic situation in the work context we observe an attempt of instrumental transfer from a different context. This transfer has characteristics of an instrumental jump from one state to another and vice-versa. From the probing questions that we asked during the project we made the following realization. Unlike student M3, who rejects the idea due to his knowledge that Geometer's Sketchpad does not have Logo-based characteristics, student M2, who is not fully aware of this distinction, dares this mental jump and tries to find ways of adjusting his idea. However, at this point the turtle idea is abandoned, at least for the time being.

The different way to start the activity indicates the two different frameworks that provide the springboard for discussion. Student M2 suggests using the turtle and expects the software to have similar functionality with the logo-based environment, while student M3 bases his reasoning [4] on the geometric object of the segment the construction of which he seems to know. This latter idea prevails for the time being and the students work on it.

**Discussion of the rest Episodes**

Then the group of boys work on the idea of student M3 for the construction of a random segment and try to give it the length of 11.3 cm with suitable manipulations. Their successful effort allows them to proceed with the same rationale to the construction of a closed line consisting of four consecutive variable segments, which resembles a rectangle that corresponds to a part of the schoolyard blueprint.

So, student M3 uses the artefact of the variable segment, a basic element of the work environment, and suggests stabilizing its length through trial and error dragging processes. His active involvement in both groups leads to the appropriation of his idea by the other students. In this way his idea becomes meaningful as it is transformed into an instrument through its collective use for the achievement of the common goal. However, two female students, M4 and M5, have not been involved in the debate so far, but carefully followed the discussion of their peers. The teacher moves a point that represents a corner of the schoolyard and the graph loses the shape of the rectangle. The students’ surprise is coupled with the reminder that the goal is to design a dynamic plan that retains its geometric properties. “*But how are we going to do it, Sir? So much work to be wasted!*” student M1 reacts. The reminder of the dynamic floor plan is the cause for a new round of talks, which last
for an hour. "I found it, sir!" student M3 enthusiastically cuts in. Going on to a
different screen he displays the vertical axes of the software and thinking aloud or
addressing the others he explains: "I'll create the first dimension on the axis using
the scale and I'll base all measurements on this length, here, when I change this
point that other one changes too".

Student’s M3 idea incorporates both the preservation of the geometric properties
of the shape and the potential for dynamic manipulation of the shape. The problem is
that he does not know how exactly to put these specific points on the axis and
demands that the dynamic geometry program provides a command corresponding to
the logo-based environment.

“What do I say, Sir?”, he asks. “Set position [11.3, 0]” He had already used this
Logo-based command in the first phase of the research when constructing a digital
puzzle. At this point we intervened and provided information for the resolution of the
specific problem within the dynamic geometry environment. Here we can observe an
instrumental jump made by student M3 during the process of instrumentation
concerning the depiction of a point on a system of axis in order to achieve the goal of
the dynamic blueprint. After that, student M3 corresponds point B of the plan, as
shown in figure 1, to point (0, 0) and works on his own with the tool "plot points"
thus completing part of the schoolyard. On seeing that student’s M3 request for an
equivalent tool to depict points in the Sketchpad environment was satisfied student
M2 repeats his question: “How can we move point A, 7.3 cm to the left, is there a
command?” This time we better evaluate the student’s question and realize that, on
the one hand, the simulation of the turtle movement command is possible while, on
the other, such a simulation requires concepts relevant to geometric transformations.
It is a pivotal point in the on-going process of instrumental orchestration as we
experience a case of instrumental genesis. The Logo-based turtle movement
command turns into a transformation instrument in the dynamic geometry
environment. Then the discussion revolves around tools of parallel transfer
transformation, which are embedded in the dynamic geometry environment.

The new tools appear to be also accepted by the two girls (M4, M5) slowly getting
involved in the drawing of the plan. While the drawing is taking place for the third
time, student M3, who has so far managed to draw a dynamic plan, addresses those
who have worked with the tool of parallel transfer with the words: “But yours
doesn’t zoom in and out! ha! ha!” Indeed, the students have designed the school
playground with accurate measurements and without moving the components
affecting the plan. However, they have not given their plan a dynamic character. This
idea opens new rounds of talks completed in two consecutive sessions. The focus is
on minimizing the measurements as well as on trying to find proportional relations
between the sides. "Let’s find all the other distances from segment 11,3”, student M2
characteristically says. Using paper, pencil and a calculator the rest of the students
support student M2 and through discussions and individual actions they conclude to proportional relationships. Due to space constraints of the present paper these discussions are not presented here despite their interesting indications of how students perceive the proportional change in shape when they work with combined modeling tools. This area has been adequately illuminated by several other researchers (Noss & Hoyles, 1996; Psiharis & Kynigos, 2004).

Students’ discussions and actions highlighted two new ways of working for the drawing of the dynamic plan. The first way adopted by student M2 required the proportional relationship between the sides. To construct the plan he used only one variable (variable-length segment) relating all the lengths to it. It should be noted that the specific student had worked on Microworlds pro in the first phase of the research in a group activity to design and present something that interested them. When his turn came he said: "I will not show you something but I’ll tell you one way I found to make one and the same shape bigger or smaller without having to write a new program". His presentation was, in fact, the structure of a parametric process, which he passionately believed he had invented himself. The reference to this fact is to indicate that the student had learned how to minimize the variables of a parametric shape in a logo-based environment and functionally transferred this knowledge to the environment of dynamic geometry software. The second mode supported by the students M4, M5 involved the multiplicative relationship between each measurement and a number corresponding to the scale factor. The probing questions showed that this method of the female students stemmed from their engagement with the concept of scale in the geography lesson. At this point they could choose one of following three ways to work: based on a scale or on a proportion or on co-ordinations. Students M4 and M5 cooperated to a sufficient extent in the design of the floor plan of the school, but student M5 was the one who completed the plan after a week. Figure 2 shows the final draft of student M5 in which point M on the top left represents the slider of the rescaling. Students M2 and M3 worked at different workstations, each one developing his plan for a while but soon abandoned the task finding it rather tedious to repeat something that they already knew. Careful study of the students’ M4 and M5 blueprint produced by the use of the tool “object properties” that is provided by the software as well as the probing questions that followed showed us that the draft floor plan was drawn with a mixed approach. They based their design on the transformations tool but they also used Euclidean geometric constructions whenever they thought they would proceed faster.

There is particular interest in student’s M5 effort to name places on the blueprint in a dynamic way. "Sir, I would like the names of places on the plan to change size along with the plan. Look I wrote Schoolyard and when the size of the plan grows the word remains the same", she explains. We were impressed by the fact that although she had repeated the same scale structure to construct the plan components dozens of
times she seemed not in need of it in this case. With more profound questions we realized that she firmly believed that if the word was in a closed-dynamic shape it should also zoom in and zoom out along with the plan. She mentioned that the word was on "a bordered area", implying the restricted area of the schoolyard. This restrictive view of the student derived from the instrumental jump she attempted based on her knowledge of the use of select and zoom buttons available in widely used software.

CONCLUSION

In the two work groups which we could monitor very closely during the set meetings while intervening when we saw fit, we observed through the practice of students the interweaving of different mental schemes, technical knowledge and specific mathematical knowledge, which determined the progress of task completion. These mental schemes became an object of negotiation. The different types of tools we chose gave the students an opportunity to mentally jump from one instrumental environment to another thus shaping both individual and collective actions. These mental passages led to the creation of mixed instrumental approaches. Knowledge transfer was possible in some cases while in others it posed a hindrance, as shown by the brief presentation of the episodes. The conditions under which it is possible to transfer mathematical knowledge from one micro-context of practice to another as well as the role of social interaction are considered attractive research topics that deserve further investigation.

NOTES


2. line 408,410

3. line 416

4. line 409,415

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