MATHEMATICS LEARNING THROUGH THE LENSES OF CULTURAL HISTORICAL ACTIVITY THEORY AND THE THEORY OF KNOWLEDGE OBJECTIFICATION [1]

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This paper draws upon analysis of an adult student as he learns to read fractions-of-an-inch on a measuring tape with the researcher as tutor. This provides a basis for comparing two activity-theoretical perspectives: cultural historical activity theory as developed by Yrjö Engeström and his collaborators and Luis Radford's recent elaboration of activity theory, the theory of knowledge objectification. The former perspective provides a general framework for analyzing activity as a whole, whereas the later draws attention to particular genetic aspects of the mathematics learning and thinking of individuals. Insights gleaned through the use of each theory in analysis of this activity are summarized in-turn and contrasted for the purpose of showing the utility of each approach for the analysis of mathematics learning.

INTRODUCTION AND THEORETICAL FRAMEWORKS

Activity theory is a cross-disciplinary framework for examining how humans purposefully transform natural and social reality, including themselves, as a culturally and historically situated, materially and socially mediated process. Originating in the dialectical socio-cultural psychology of Vygotsky, this work was subsequently developed into a theory of activity by his student and colleague, A. N. Leont'ev (cf., 1978) and others. Today in the west this perspective is often associated with the work of Engeström and his collaborators and is referred to as cultural historical activity theory or the acronym CHAT, emphasizing the essential situated nature of activity. It continues to develop in different ways, highlighting both its complexity and the fact that CHAT remains, in many ways, a work in progress.

The impromptu tutoring session involving a pre-apprentice in the pipe trades and the researcher that formed the basis of this analysis focused on learning to read fractions-of-an-inch to sixteenths-of-an-inch; an essential skill in this training program and the workplace. The present paper uses an analysis of this workplace training course as a whole as well as an in-depth analysis of the targeted tutoring session using both Engeström's interpretation of CHAT and Radford's theory of knowledge objectification (TO). This, in turn, provides a basis for comparing and contrasting these two perspectives. This analysis is part of a larger study of mathematics learning within apprenticeship training conducted in a number of pre-apprenticeship and apprenticeship training programs in the construction trades.

A brief overview of Engeström's interpretation of activity theory

Engeström's work develops the implications of Leont'ev's ideas and systematizes them in the form of an activity system. (Because of space restrictions, a figure showing Engeström's well known triangular activity system model will be provided only in the results section.) Included as the co-mediating elements within an activity system are the subject, community, tools (including signs and artifacts), rules or norms, and division of labour; all oriented towards the object and outcome of the activity (cf., Engeström, 1987, 1993, 2001). And, for any activity system, it is the meeting of a human need that motivates the activity.

Engeström also articulates a number of key principles of activity. The following of these are most pertinent to the present assessment of this perspective for the analysis of mathematics learning:

• that a collective, artifact-mediated and object-oriented activity system, seen in its network of relations to other activity systems, is taken as the prime unit of analysis…;

• [that] an activity system is always a community of multiple points of views, ... and interests….;

• [that] activity systems take shape and get transformed over lengthy periods of time. Their problems and potentials can only be understood against their own history...; and

• the central role of contradictions as sources of change and development [of an activity], …; (Engeström, 2001, pp. 136-137).

The larger scale view of activity provided by this perspective considers learning in terms of fundamental qualitative changes in an activity system as a whole, a process that Engeström calls expansive learning. This occurs as a result of deliberate efforts by participants over time to resolve inherent conflicts and contractions that are an inherent part of any activity system. Engeström's theorization provides little, if any, explicit direction for understanding the place of mathematics within activity nor provides details relating to the learning process of individuals. He does, however, acknowledge the need for context-specific concepts and methods to be created and employed when applying CHAT to specific empirical cases (Engeström, 1999, 2008).

A brief overview of Radford's theory of knowledge objectification

Based on his reading of Vygotsky's semiotics, Leont'ev's activity theory, and the more recent work of Felix Mikhailov and Evald Ilyenkov, Radford has developed the TO specifically for unpacking the nuances and processes of the mathematics activity and learning of individuals from a cultural-semiotic activity perspective (cf., Radford, 2006, 2007, 2008b). In contrast to Engeström, Radford's work focuses on specific aspects of the consciousness, learning, and being of individuals as well as the semiotic and social dimensions of mathematics activity from an activity perspective. Foremost in Radford's theorization are emphases on:

1) the intimate and dialectical relationship between human thinking—including mathematical thinking—and the material and cultural world,

2) the central role of semiotic systems used within culturally and historically bound practices and social interaction in mathematical activity and learning, and

3) the twin dialectical processes of *subjectification*—the process of becoming, and *objectification*—the process of making sense of and becoming critically conversant with the cultural-historical logic with which systems of thought, such as mathematics, have been endowed (see also Radford, 2008a, 2009);

In the TO, learning is conceptualized as an interactive and creative acquisition of historically constituted forms of thinking. Such an acquisition is thematized as a process of *objectification*; that is, as a process of making sense of and becoming critically conversant with the cultural-historical logic with which systems of thought, such as mathematics, have been endowed (see also Radford, 2008a, 2009). Radford's concept of objectification is a refinement of Vygotsky's notion of internalization in that it emphasizes the dialectical relationship between the subject and the cultural object being attended to. *Semiotic means of objectification* (SMO) is the empirical reflection of this process. This refers to the use of semiotic means to draw and sustain the attention of others and one's own attention to particular aspects of mathematical objects in an effort to achieve stable forms of awareness, to make apparent one's intentions, and/or to carry out actions to attain the goal of one's activity. Radford identifies the following three processes as forms of SMO from his empirical research of collaborative mathematical problem solving and learning:

1) *Iconicity*—the process of noticing and re-enacting or re-voicing significant parts of previous semiotic activity for the purpose of orienting one's actions and deepening one's own objectification (Radford, personal communication, September 29, 2008),

2) *Semiotic nodes*—places in mathematical activity where multiple semiotic systems are used together and in a coordinated manner to achieve knowledge objectification. "Since knowledge objectification is a process of becoming aware of certain conceptual states of affairs, [changes in] semiotic nodes are associated with the progressive course of becoming conscious of something. They are associated with layers of objectification" (Radford, 2005), and

3) *Semiotic contraction*—the process of coming to recognize and attend to the essential elements within an evolving mathematical experience; and making one's semiotic actions compact, simplified, and routine as a result of this acquaintance with conceptual traits of the objects under objectification and their stabilization in consciousness (Radford 2008a).

It should be noted that the process of mathematics learning or objectification can be accounted for readily in the theory of knowledge objectification through analysis of social interactions and the use of semiotic means of objectification within mathematics learning activity (Radford 2008b).

METHOD

Data collection

The empirical data that provides the basis for the present analysis was collected in a pre-apprenticeship training class for the pipe-trades conducted at trade union run school in British Columbia, Canada. This program involved classroom work as well as practical work in the shop. The course content was selected to give the pre-apprentices a head start with important skills that would be addressed subsequently in the early years of their formal apprenticeship training in a number of different pipe-trades.

The researcher visited the class extensively over its eight-week duration and served as a math tutor for any pre-apprentices who asked for his help. The researcher also observed pre-apprentices and engaged them in discussion about their mathematics related coursework as they were working on it. The mathematics related activity of individual and groups of pre-apprentices, working either on their own or with the researcher, was documented using a video camera and field notes, and copies of the course print materials and the pre-apprentices' written work were retained for analysis. The researcher also conducted ongoing formal and inform discussions with the course instructor and administrator of the program, as well as many students and kept field notes of these encounters. The data used in the present analysis was drawn from this collection of data. This particular tutoring session was selected for fine-grained analysis from the approximately 35 hours of video data from this pre-apprenticeship class on the basis that it was, by far, the longest episode of a student focused on a particular mathematical object that was central to this vocational training. This, in turn, provided a unique opportunity to examine ways in which a pre-apprentice's thinking developed in relation to this mathematical object and was reflected both in his actions and in his interactions with the researcher.

Prior to the data analysis (as part of the larger research study involving multiple workplace training sites) the researcher made extensive visits to an earlier session of this same pre-apprenticeship course as well as a fourth-year plumbing apprenticeship program at a local technical college. Extensive visits were also made to all three levels of an iron-working apprenticeship program also at the local technical college and short visits spanning one to a few days were made to a variety of other apprenticeship programs for a variety of construction trades. In all cases the focus was on observing and documenting the mathematics related parts of these programs. This experience served to inform the researcher's analysis of the activity within the target pre-apprenticeship program.

Data analysis

The multi-semiotic analysis of this activity takes place on two levels, reflecting the breadth of foci of the two theoretical perspectives used. The various elements of the activity system of the pipe-trades training program, for example, were discerned from the entire corpus of related data identified above. At another level the multi-semiotic analysis of the pre-apprentice's and the researcher-as-tutor's joint activity during the tutoring session began with the construction of a verbatim transcript of the entire tutoring session from the video recording. This included the construction of a detailed account of significant actions including the use of semiotic systems, other utterances, gestures, body position, and artifacts. Episodes in the data were then coded, first to identify the various aspects of reading the fraction-of-an-inch pattern being attended to, and then to identify actions related to both the pre-apprentice (who will henceforth be referred to as "C") and the researcher-as-tutor's (who will henceforth be referred to as L) objectification of the pattern of fractions of an inch on the measuring tape and subjectification as participants within this activity. At times this process required slow motion and frame-by-frame analysis of videotape to assess the role and co-ordination of spoken language with the use of artifacts and gestures.

RESULTS AND DISCUSSION [2]

A summary analysis using Engeström's interpretation of activity theory

A detailed analysis of the elements of the tutoring session activity system is provided using a triangular activity system model in Figure 1. The lines between the nodes or elements of the activity are intended to draw attention to the mediating relationships amongst them—an essential feature of such a system. While it is not possible to determine the precise mediating roles of each of these elements throughout the activity, there is evidence of each playing a dynamic role in shaping the course of events. The various semiotic systems employed, for example, serve to draw C's attention to particular aspects of the object of the activity and to deepen his understanding. The design of the particular measuring tape used (marked in thirty-seconds-of-an-inch up to twelve inches and in sixteenths thereafter) necessitated that this difference be attended to explicitly and negotiated during the activity. And, the conventional design of the measuring tape with the endpoints of subintervals of the inch indicated by a system of signs necessitated that C attend to the intervals between these divisions rather than the division markings themselves in the process of learning to measure.

A number of contradictions in the form of misalignments, breakdowns, misunderstandings, and complications exist on different levels within this activity. These levels include complications within individual elements of the activity such as the object of the activity itself—the system of multiple binary fractions-of-an-inch represented by a single inscription pattern on the measuring tape—and with L given his dual roles as tutor and researcher. Between different elements of the activity there are a number of other misunderstandings and breakdowns. These include C's unfamiliarity with the imperial system of linear measure that he is required to use in his practical work, his limited understanding of the meaning of the denominator of a fraction represented in digit form at the start of the session, C and L's communication difficulties, and L's initial relationship with the norm in the pipe-trades of measuring

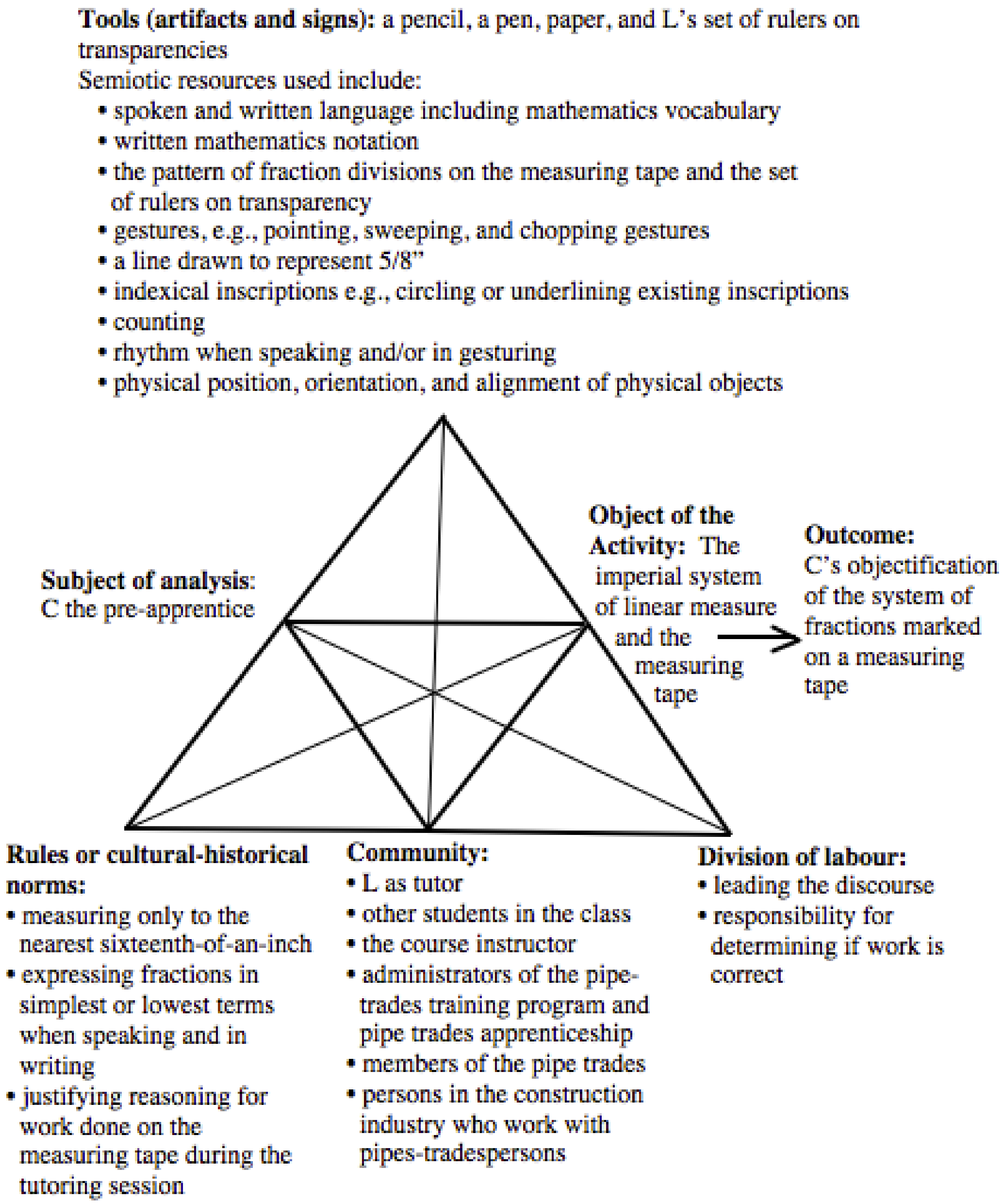

Figure 1. The activity system of the pre-apprentice learning to read the measuring tape

only to sixteenths. Considerable effort is made by both participants throughout the tutoring session to attend to and resolve these contradictions as they arise within the activity.

A summary analysis using the theory of knowledge objectification

A number of processes that are parts of the mathematics learning process identified by the TO figure prominently within C and L's discourse. For example, C repeats what L says or re-enacts his actions relating to the task at hand on 60 separate occasions during the 33 minute tutoring session. These actions reflect an effort to deepen his sense of—literally, to deepen his sensory experience of—these statements or actions using the same means of semiotic expression used by L or other means. On one occasion C re-enacts a unique form of interval gesture that he had just used himself and on another occasion L creates a zone of proximal development for C to help bring coherence to his understanding of the division pattern on the measuring tape by inviting C to explain what he (L) had said earlier and then by providing him with verbal prompts to help him along. And, this is not a one-sided affair. On a few occasions L also re-enacts or repeats what C had done or said earlier. These examples of repeating or re-enacting what another has said correspond to the process of iconicity, identified by Radford as a significant part of the process of attaining a cultural logic of thinking or knowledge objectification. [3]

The ways that L and C use multiple semiotic systems together (semiotic nodes) throughout the tutoring session and, in C's case, the further enactment of semiotic contractions, reflect their understandings of the object of the activity. L's frequent use of various combinations of words, pointing and sweeping gestures, fractions written using digits and words, the fractions-of-an-inch division pattern on the measuring tape and transparency rulers, along with other semiotic resources in a coordinated manner draws and maintains C's attention to/on various aspects of the system of binary fractions-of-an-inch on the measuring tape. And, given L's extensive experience working with the system of binary fractions-of-an-inch extending back to his own elementary school days, it is not surprising that his use of various semiotic systems remains relatively consistent during his explanations to C throughout the tutoring session, reflecting little or no change in his understanding in the process.

In contrast, there is a marked shift over the 33 minutes of the tutoring session in the way that C expresses his understanding using various combinations of semiotic systems as he communicates with L and works to bring clarity to his own thinking. Early on, when C responds to L's request for him to explain what difference he notices in the patterns of divisions below and above twelve inches on the measuring tape, C's response is predominantly gestural, accompanied by only a single sentence and two sentence fragments. As the tutoring session progresses, C's means of expressing himself shifts completely, at times, to the clear and succinct use of words alone—an ultimate form of semiotic contraction.

The last process from the TO to be summarized here is that of C and L's subjectification. C becomes more active in the way in which he participates in the activity over the duration of the tutoring session. This is evidenced by the collective changes in the patterns of his gaze and attentiveness, his role in the dialogue, his affective responses, and his own expressions of agency and self-reliance regarding his use of the measuring tape. C also nods his head or says “okay” or “yeah” on numerous occasions throughout the session acknowledging to L that he is following what he is saying. This also reflects part of C’s process of subjectivity within the activity. L changes during the tutoring session as well, but in a less obvious way. Specifically, L changes in his approach to teaching C how to read the measuring tape from a more generalizable approach (intended for any form of binary measuring tape or ruler) that is typical of school mathematics teaching, to a much more practical one tailored specifically to the workplace demands within the pipe trades.

Comparing these two activity-theoretical perspectives

Engeström’s activity-theoretical perspective focuses on activity systems as a whole, including their ongoing transformation over long periods of time. And, his more recent work continues at the activity system level focus by addressing interactions between multiple activity systems (e.g. Engeström, 2008). Engeström drives this point home with his assertion that the activity system is the primary unit of analysis. His attention to contradictions at various levels of activity and to multiple points of view within any activity provide ways to interpret aspects of activity at other levels as well. These foci, while useful for research in many contexts, run counter to mathematics educators’ practical interests in teaching and learning activity, that is, individual students’ mathematical enculturation on a day-to-day, if not minute-to-minute basis. Furthermore, Engeström’s work does not make clear a means to talk about, nor situate mathematics within activity.

Radford, on the other hand, is a mathematics educator and his development of activity theory and his empirical research reflect a need to understand the mathematics learning processes of individuals. He provides a way of defining and positioning mathematics as a cultural practice within particular forms of activity, articulates a clear view of mathematics learning and thinking, and unpacks the dialectical relationship between the subject of activity and the object of activity through his theorization of objectification and subjectification thus revealing this important but often tacit dimension of mathematics learning. (For a detailed discussion refer to Radford, 2008b.) In stark contrast to Engeström who theorizes learning within activity theory as change in an activity itself, Radford focuses on the learning of individuals as they come to be part of an existing historical-cultural activity.

CONCLUDING REMARKS

The analysis of a one-on-one tutoring session with a pre-apprentice learning to read a measuring tape summarized here using both Engeström’s perspective on CHAT and Radford’s theory of knowledge objectification illustrates differences between these two approaches for the analysis of mathematics activity. Clearly both of these activity-theoretical perspectives have contributed and will continue to contribute to mathematics educators’ understanding of mathematics activity and learning. But given the objects for which each was developed, on the one hand Engeström’s theorization for activity in general and, on the other, Radford's theorization for mathematics learning activities in particular, it should come as no surprise that Radford's theory of knowledge objectification provides a much more powerful and directly applicable tool for investigating and understanding mathematics learning. In pragmatic terms, before embarking on any activity-theoretical analysis in mathematics education, it is advisable for researchers to consider and draw from a range of activity- theoretical perspectives given that this field is evolving in a variety of different ways.

NOTES

1. This paper is the result of a research program funded by The Social Sciences and Humanities Research Council of Canada / Le Conseil de recherches en sciences humaines du Canada (SSHRC/CRCH).

2. Given the focus of this paper on comparing different activity-theoretical perspectives and space limitations, only a summary can be provided of the analysis done using each perspective.

3. A detailed analysis of the different forms of iconicity evident within a single 21 second clip from this tutoring session was the focus of a paper presented at CERME-6 (see LaCroix, 2010).

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