RESEARCH PRAXEOLOGIES AND NETWORKING THEORIES

M. Artigue,¹ M. Bosch,¹& J. Gascón³
¹Université Paris 7 (France), ²Universitat Ramon Llull (Spain), ³Universitat Autònoma de Barcelona (Spain)

Abstract: In this contribution, we consider the potential offered by the anthropological theory of the didactic (ATD) for addressing the issue of networking between theories through the extension of the notion of praxeology, which is at the core of ATD to research praxeologies. After introducing such an extension, we discuss its implications in terms of networking, giving a crucial role to the notion of didactic phenomena. We then use this language for reflecting on two networking experiences in which we have been involved.

INTRODUCTION

In accordance with the work done by the ‘theories group’ in the last two CERMEs, this contribution joins the efforts made to support a productive cooperation in European mathematics education research, in order to overcome the framework compartmentalization that could hinder the capitalisation of knowledge and its practical exploitation. These efforts have shown that the interaction between researchers working with different approaches has to go further than the ‘communication paradigm’ that dominates exchange activities in most international conferences. It needs real ‘teaching and learning’ activities to explain what one does and to understand what the others do. Experiences in ‘networking theories’, carried out in this sense¹, have shown that their productive development also requires the consideration of a shared epistemological model, that is, a common way of thinking and talking about what scientific work is and how it evolves. In fact any research activity supposes a particular implicit way of interpreting the nature of problems that are approached, the empirical field to consider, the kind of methodologies that can be used and, more generally, what research is and what it is for. When the exchange between researchers attains the level of the theoretical bases – as is the case in the ‘networking theories’ activities –, then it becomes necessary to question the implicit epistemological model of each approach, looking for a common language to express and discuss the respective epistemologies. In this sense, the Anthropological Theory of the Didactic (ATD) that we use in our research and, more concretely, its central notion of ‘praxeology’ has progressively appeared to us as a useful tool to develop such a common ‘language’ or epistemological model. Here we present the recent work we have undertaken in this direction.

¹ We refer more especially to the so-called ‘Networking group’ led by Angelika Bikner-Ahsbahs that emerged from CERME4 and to the work carried out by the first author in European projects like TELMA and ReMath.
‘THEORIES’ OR RESEARCH PRAXEOLOGIES?

If, according to ATD, we assume the general anthropological postulate that all human activities can be described in terms of ‘praxeologies’ (Chevallard, 1999, 2006), this must be also the case for research activities. Any kind of research, including ‘networking’ projects, should thus be subsumed under the notion of research praxeologies. In this perspective, talking about ‘theories’ (as in the expression of ‘networking theories’) is the result of a metonymy used to point to the whole – research praxeologies – by only indicating one part, the theoretical block of praxeologies. As any other praxeology, research praxeologies are indeed composed of an amalgam of pieces that can be described by a set of four elements \([T/\tau/\Theta/\Theta]\). The pair \([T/\tau]\) corresponds to the ‘practice’ (or know-how) of research, with the types of problems \(T\) that are approached and the techniques \(\tau\) used to approach the problems. The block \([\Theta/\Theta]\) forms the technological-theoretical discourse used to describe, justify and interpret both the research practice and the results obtained. This theoretical block corresponds to research ‘knowledge’ and is often considered as the representative of the whole praxeology, with the limitations and biases that this reduction can generate in the approach to and treatment of ‘networking’ issues.

We postulate that the notion of praxeology can help overcome these limitations and that it can also be useful to retrospectively reflect on networking efforts. We also find it important to stress that research praxeologies, as any other praxeological form, are ‘alive’ entities that evolve and change, which affects at the same time their four components \([T/\tau/\Theta/\Theta]\) and the interaction of these. The evolution of the practical block \([T/\tau]\) produces new theoretical needs that make the theoretical block \([\Theta/\Theta]\) progress and, reciprocally, the evolution of concepts, interpretations or ways of thinking and the emergence of new results lead to the construction of new techniques and the formulation of new problems. Research praxeologies can appear as different kinds of amalgams, more or less organised depending on the maturity of the field. It is the historical development of the field that helps structure these praxeological amalgams, making them more coherent and easier to disseminate according to different didactic and institutional transposition processes. Beyond the static description of research praxeologies in terms of their practical and theoretical blocks, processes piloting their dynamics are still to be analysed in depth. Our contribution consists in considering the notion of ‘phenomenon’ and relating it with the ‘technological’ element of praxeologies, which will highlight its crucial role in the dynamics of praxeologies.

‘PHENOMENA’ AND THE DYNAMICS OF RESEARCH PRAXEOLOGIES

The notion of ‘didactic phenomenon’. Today the notion of ‘phenomenon’ does not happen to have a central function in many didactic approaches. It did however play a crucial role in the emergence of the theory of didactic situations (TDS) and its vision
of didactics as a scientific discipline. In the first developments of TDS in the 1980s, and through several different formulations, Guy Brousseau (1997) defined didactics of mathematics as the science the essential aim of which is the knowledge of didactic phenomena, that is, phenomena appearing in the teaching, learning or, more generally, diffusing of mathematical knowledge in social institutions (including school ones). Hence, didactic phenomena have to be considered both as a construction and as an object of study of didactics, in the same way that physics studies the specific construction ‘physical phenomena’, or sociology studies and also defines social phenomena, etc. –including all the historical controversies about phenomena delimitation in nature and social sciences.

What is the role played by phenomena in relation to research praxeologies and their evolution? In a first approach, we can characterise didactic phenomena as empirical facts, regularities that arise through the study of research problems. Some of these phenomena enrich the initial theoretical framework to produce new interpretations and techniques or research methodologies, while others remain at the level of ‘results obtained’ and are reinvested to formulate new problems or to propose new diagnostic and practice-development tools. In order to clarify the relation between the notion of phenomenon and the four components of a praxeology, let us start from a very simple example of mathematical praxeology. Let us consider Pythagoras’ theorem or, to be more precise, the phenomenon underlying this theorem, that is, a certain regularity between the measures of the sides of right triangles. At the beginning we can consider a type of mathematical problem that could be formulated as the problem of the characterisation of a right triangle or the graphical representation of a right angle. The answer to this problem appears as a technological element (the description of a property of a given set of figures) within the mathematical praxeology that emerges around this type of problems. This technological element is not only the description of a regular fact: it also produces new mathematical techniques, helps formulate new problems and discover new regularities, thus producing more technological results. In the long run, if the initial regularity appears to be strong enough, then it comes to integrate the theory of the praxeology as a basic principle of certain kind of geometries (those with a Euclidian metric).

It is thus an entire mathematical praxeology, with its types of problems, its techniques and its technological-theoretical discourses that the expression ‘Pythagoras theorem’ refers metonymically to. Behind a technological ingredient such as a theorem – or any other description of a regular fact or phenomenon – we can find a whole set of praxeological ingredients (problems, techniques, etc.), which this technological ingredient contributes to structure. Taking all necessary precautions, we will briefly establish a parallelism between this example and research praxeologies in didactics. We will use a concrete example, the phenomenon
of didactic transposition, to illustrate our proposal\(^2\) which is to see the role played by phenomena in the ‘structuring’ of praxeologies, that is to say in their dynamics.

**Phenomena and type of problems.** As any scientific discipline, didactics of mathematics aims at identifying and studying a specific kind of phenomena (didactic ones) in order to attain a greater capacity of action and comprehension. Any research question or problem thus has to be related – even if this relation is mostly done a posteriori – to the highlighting of a phenomenon, its delimitation, the conditions needed for its existence and evolution, etc. Even if research takes as a starting point a problem emerging from a very concrete teaching or learning practice (as it often does), an effort is always made to formulate the problem in a more general way, implicitly considering it as a specimen of a given ‘type of problem’. This is a first step in the work that follows: looking for regularities related to the practical issue approached, trying to characterise them and, to some extent, ‘understand’ or ‘act upon’ them. Let us consider, for instance, the phenomenon of didactic transposition as it was characterised by Yves Chevallard (1985). Several new problems have been raised and studied that could not even have been formulated before the identification of the phenomenon (see Bosch & Gascon, 2006 for a recent review).

**Phenomena and technological components** In research processes, the results obtained as an answer to the raised problems generally contribute to enrich the initial research technology by integrating new characteristics of the studied phenomena or even new derived phenomena. There always exists a double-direction effect between the results obtained and the evolution of the technology of research praxeologies, which can be considered at the core of progress of scientific research.\(^3\) For instance, the study of transpositive processes in different mathematical domains has highlighted various phenomena that, in turn, have been used as a starting point to formulate new problems and draw attention to new regularities. A good example is the phenomenon of the ‘algebraisation’ of Calculus at upper secondary school level (Artigue, 1995), a result that has then been used to analyse the teaching of limits of functions (Barbé et al., 2005). Other examples coming from the didactic transposition processes are the derived phenomenon of the ‘stoppage’ of didactic transposition (Assude, 1993) or of ‘detransposition’ (Antibi & Brousseau, 2000).

\(^2\) We are perfectly conscious of the distance between mathematics and didactics as fields of research. However, commonalities can be established and can be productive in both senses: sometimes the maturity of mathematics hides some evolution phenomena that are more visible in the recent and less developed dynamics of didactics.

\(^3\) This is less true when the theoretical block of the research praxeology comes from a different discipline. We then obtain a single-direction effect which ‘breaks down’ the dynamics of scientific research: for instance, when a given notion of cognitive psychology is used to analyse some facts related to the learning of mathematics, because the ‘external’ character of the results obtained, they will have no effect on the development of the initial psychology notional frame.
Phenomena and technical components The study of phenomena not only generates the description of regularities, restrictions or ‘paradoxes’ (like those of the ‘didactic contract’, for example). It also leads to new ways of doing research, that is, new techniques and new methodologies. In the case of transpositive phenomena, the highlighting of relations and differences, both chronological and diachronic, between the ‘scholarly knowledge’, the ‘knowledge to be taught’ and the ‘(actually) taught knowledge’ has now become a technique of didactic analysis by itself. Almost any problem studied within the ATD or the TDS includes, to some extent, a questioning about which is the knowledge at stake, where does it come from, what ‘scholarly knowledge’ legitimises its teaching, what changes have been operated on it, what ‘noospherian’ discourses support or hinder its teaching, etc. The notion of didactic transposition has represented an important enlargement of the field of study of didactics because it has pointed out the need to also consider the mathematical activities that exists outside the school (Bosch and Gascón, 2006).

Phenomena and theoretical components. In a praxeology, the ‘theory’ component includes the set of notions and relations that are used to apprehend phenomena (describe them, formulate questions about them, etc.), to develop them and to identify new regularities. The ‘theory’ appears as the second level of validation of the activity, as an explanation and justification of the ‘technology’. It contains the assumptions taken, that is, the technological elements that come up being taken for granted because of their solidity and persistence. At this level we find questions such as: What phenomena are studied? What is a problem in didactics? Why can this or that result be assumed as such? The empirical enlargements mentioned before are also integrated at this level as far as they become basic and implicit assumptions. At the same time, the unit of analysis that is assumed determines the kind of phenomena that can be considered and the kind of data that are being collected to bring evidence to the study. For instance, the existence of transpositive processes between institutions is a theoretical assumption that is not questioned, nor questionable, in ATD. The ‘kind’ of transpositive processes that are taking place, their main characteristics and the conditions and restrictions they create on teaching institutions are, on the contrary, some of the main problems considered by this approach.

The praxeological dynamics we just described may help understand the processes through which the studied phenomena produce new technological results that partially become new theoretical tools and produce in turn new research techniques allowing the identification or construction of new phenomena. It is this praxeological dynamics that we propose to consider here in order to analyse – and guide –two networking experiences between European research teams.
THE EXPERIENCE OF THE GROUP «NETWORKING THEORIES IN MATHEMATICS EDUCATION»

The working group on ‘networking theories’ was created in 2005. It includes 12 researchers from six different countries and its work aims at the exchange, comparison, and connection between theoretical frameworks. Results obtained have been presented in previous CERMEs and more recently in a research forum at the last PME conference (Bikner-Ahsbahs et al., 2010). In this section, we analyse an episode of its work, already evoked at CERME6 (Artigue, Bosch, Gascon & Lenfant, 2010), with the aim of showing how research questions and theoretical components of praxeologies influence research methodologies (technical components), the units of analysis considered pertinent, and the didactic phenomena identified.

The initial work of the group was based on a video realized in a grade 10 Italian classroom and additional material considered necessary for its analysis by our Italian colleagues. Each team was asked to analyze the video from its own theoretical perspective but the data provided was judged insufficient by each except the Italian team. The video showed two students working in a pair, with little intervention of the teacher. Additional information provided on the session itself and its context was quite limited, making an analysis supported by TDS very hypothetical and an analysis supported by ATD nearly impossible. A questionnaire was then addressed to the teacher in charge of the classroom, asking for additional information to allow the different teams to complement the partial analyses already carried out. In the teacher’s answers, the attention of several members of the group was especially attracted by the following excerpt:

*I try to work in a zone of proximal development. The analysis of video and the attention we paid to gestures bring me to become aware of the so called ‘semiotic game’ that consists in using the same gestures of students but accompanying them with a more specific and precise language in a relation to the language used by students. Semiotic game, if it is used with awareness, may be a very good tool to introduce students to institutional knowledge.*

This convergence of interests led the group to develop a new strategy for progressing in the collaborative work undertaken: The TDS team should associate a question articulated in the TDS framework to this excerpt, and then each of the other teams should rephrase this question according to its own perspective. We reproduce below the text introducing the TDS question, which in fact also uses some ATD constructs.

*The connection between the mathematics produced by students in what we would label, using the TDS frame, an adidactic situation through interaction with the adidactic milieu of this situation on the one hand, and the institutional knowledge aimed at on the other hand, generally requires at least changes in the ways the mathematics at stake is expressed in order to progressively tune these it conventional forms of expression. The teacher considers that he has a specific mediating role to play for making this connection possible and uses semiotic games as a tool for that. In other terms, semiotic games can be*
considered as components of the praxeology (or more certainly one of the praxeologies) that he has developed in order to solve this didactic task.

The expression “semiotic game” thus denotes what can be seen as a technique, a component of a teaching praxeology, resulting from the identification of some particular phenomena of semiotic mediation. Interpreted that way, it shows how a theoretical focus (in this case a semiotic focus) can lead to the identification of specific phenomena, and from that to theoretical constructs or to didactic techniques, considered as tools for improving the efficiency of learning and teaching processes. A TDS perspective leads to question the efficiency of such a didactic praxeology for two reasons at least. The first one is that very often interactions with the adidactic «milieu» do not guarantee the possibility of establishing a direct connection with the institutional knowledge aimed at. These limitations are the source of different didactic phenomena identified as paradoxes of the didactic contract: Topaze effect, Jourdain effect, meta-cognitive slide. The second reason is that the adidactic situations most often observed in classrooms are situations of action, not situations of formulation. In such situations, some linguistic activity generally takes place but it is not taken in charge in the piloting of the situation through didactic variables.

From this perspective, the video analysis leads to the postulate that, in this particular context, the distance between what the students have autonomously produced and the forms of knowledge aimed at by the teacher, as expressed in his answers to the questionnaire, makes problematic the productive character of such a semiotic game. Thus the question proposed to the group:

*Do the episodes at our disposal allow us to identify characteristics of the semiotic game technique that would help us to understand their potential for compensating the possible limits of the interaction with the adidactic milieu for achieving the expected mathematical goals, and linguistic evolution linked to the needs of institutionalization processes?*

In the networking group, each team has rephrased this question from its specific theoretical perspective. This episode shows how the consideration of a new theoretical framework, here TSD, can lead to question a didactic praxeology, legitimately considered as a research result in another didactic culture. For addressing this question, a new research praxeology has been developed, a research praxeology which had no reason to emerge in either of these didactic cultures and only exists because a specific networking activity has been undertaken. In the limited space of this contribution, we cannot present the results produced by this research praxeology, nor their elaboration into didactic phenomena. We nevertheless hope to have shown up to what point the relationships existing between the different components of research praxeologies and the didactic praxeologies emerging from research results, deserve our attention. We also hope to have shown that an approach in terms of praxeologies can be helpful for addressing networking issues.
THE EXPERIENCE OF THE EUROPEAN PROJECT REMATH

The experience of the European project ReMath (http://remath.cti.gr) offers complementary insights for putting to the test an approach of networking theories in terms of research praxeologies. An essential goal of this project was to support the capitalization of research on digital technologies in mathematics education, through the development of an integrated theoretical framework, with a focus on the affordances of digital technologies for mathematics learning in terms of representations, and more globally of semiotic activities. Six European teams worked on this project during four years, relying on the previous experience of the European research team TELMA (Artigue, 2009). A sophisticated methodology was developed for this project. It relied on a meta-language created in TELMA and a system of cross-experiments using the same digital technology in different didactical cultures, whose negotiation, implementation and analysis was taken itself as an object of research. This project also allowed the researchers involved to insert the networking activity into a permanent dialogue between the design of digital artefacts and the design of scenarios for their educational use in different educational contexts.

Looking back at this project through the lens of research praxeologies, it looks clear that the methodology used allowed the ReMath teams to organize their work around the collective study of their respective research praxeologies. These research praxeologies were made explicit enough in their different components for ensuring the productivity of comparative analyses, and particularly that of the cross-case studies of the different experimentations carried out with the same digital artefact. The articulation of common research questions to be addressed by the different cross-experiments and then the addition by each team of questions reflecting its specific concerns, the strict organization of interactions between teams all along the process, from the design of artefacts to the a posteriori analysis of cross-experimentations, the meta-language of concerns, played an essential role. The design of artefacts and the cross-experimentations contributed ipso facto to two different types of research praxeologies: on the one hand, praxeologies inserted in the didactic culture proper to each team and, on the other hand, a “networking praxeology” still in development. The problem addressed by the first ones was the identification of the learning affordances of the systems of representations of mathematical objects implemented in the six digital artefacts, and of the conditions for a possible ecology of these in realistic contexts. The problem addressed by the second one was networking between theories. It situated at a meta-level with respect to the first ones, and the results it produced have a different nature. Some are methodological and a priori regard more the practical block of this networking praxeology, for instance those concerning the technique of cross-experimentation, an essential ingredient of the networking praxis progressively refined. Some are more likely to contribute to its theoretical block. This is the case for the boundary objects identified for facilitating the communication between the theoretical frameworks at
stake and for the STF (Shared Theoretical Frame). Some results show the possibility of connections and even offer partial integration of theoretical frameworks while some others identify limits to such ambitions, but it is worth noticing that, at this stage, none of these results has been given a clear status of phenomenon. Due to these characteristics, there is no doubt for us that a systematic a posteriori analysis of the ReMath project using the notion of research praxeology should be helpful.

CONCLUDING REMARKS

The retrospective analysis of the two research projects outlined above seems to confirm our initial postulate that networking between theoretical frameworks must be situated in a wider perspective than that consisting of the search for connections between the objects and relationships structuring these. From this point of view, our approach is fully coherent with that developed by Radford (2008) who, defining a theory as a triplet made of a system of principles, a methodology and a template of research questions, insists on the necessity of considering these three components in networking activity. Space limits do not allow us to enter into a comparison of our approach with that presented by Radford conceiving a network of theories as a semiosphere, but we hope that the discussions in the CERME Working Group will contribute to clarify similarities and differences. In our approach for instance, the notion of phenomenon appears as a crucial notion for understanding the dynamics of research praxeologies and the evolutionary links between their different components, while the word phenomenon is absent from Radford’s text. This is an intriguing difference which certainly needs to be collectively analysed and discussed.

Our reflection tends to show that an approach in terms of research praxeologies can be productive for networking between theories, especially because it helps address the essential issue of the functionality of theoretical frameworks, by inserting these in systems of practices. Networking between theoretical frameworks, if considered as a task to be solved, requires nevertheless the development of specific praxeological elements that cannot be separated from research praxeologies. The European projects evoked above attest the existence of such elements, with emerging techniques and embryonic technologies made of classifications, structured landscapes, meta-languages. The model of praxeologies could thus help us compare the different existing efforts of networking and develop more productive ones. This is nevertheless only a hypothesis which has not yet been seriously worked out. Finally, we would like to stress that when adopting such a perspective, one must remain sensitive to the fact that this approach, as any other one, can also introduce some limitations. Considering them is indeed part of the epistemological vigilance required in any research process.

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