EXPLORING FRAGMENTATION IN MATHEMATICS EDUCATION RESEARCH
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This paper reports on a study which uses techniques from social network analysis to explore patterns of research collaboration within the mathematics education research field, focusing on English language publications. The results suggest that the perceived theoretical or methodological fragmentation in mathematics education does not translate in a straightforward way into a fragmentation between researchers. This result is used to argue for a method for connecting research practices complementary to some of those discussed in previous papers in the CERME working group. Namely, that existing collaborations could be used to promote increased theoretical or methodological coherence across the research field when combined with the strategies already developed by the group.

INTRODUCTION

One of the reasons often given for addressing the multiplicity of theoretical perspectives and approaches in mathematics education and for attempting to connect them has been to reduce the perceived fragmentation and diversity in the field (Prediger, Bikner-Ahsbahs, & Arzarello, 2008; Prediger, Bosch, Kidron, Monaghan, & Sensevy, 2009). The field has been criticised (from within and without) for lack of coherence, for failing to build on previous results and for susceptibility to fashion and cyclically reinventing old ideas (see for example references in English, 2002; Maasz & Schloeglmann, 2008; Sierpinska, 2003). The fragmentation and diversity is not just a result of lack of agreement about the theoretical framework we should work in, or even about whether we should work in one, or what role it ought to play. It can also be understood as a result of the social organisation of research and the conditions under which it is carried out, in particular, its openness as a field to political and other external influences, the different national contexts (intellectual, social, political) in which it is carried out, and the diverse educational and training backgrounds of those carrying out research. Any programme for managing the fragmentation and diversity of the field needs to draw on an understanding of the state of the field and the mechanisms which have led to and perpetuate the current fragmented state. Based on such an exploration we can recognise and develop possible mechanisms for change and ask questions about the social conditions necessary for change. Additionally, the social conditions of research place constraints on the goals which are possible in addressing the theoretical and methodological diversity of maths education research. An awareness of this adds an important dimension to discussions of what mathematics education research as a field should aim to be, or how it ought to develop.
Discussions in the papers of the CERME working group on theoretical diversity in the European mathematics education research community and ZDM special issue 40(2) have made a move from the abstract discussion of principles etc. to more concrete attempts to connect theories and this move has been accompanied by questioning of the different ways in which theory can be defined, and (to a lesser degree) the nature of theory and theory/practice relationships. In particular, the idea of theory has been developed and expanded by a number of authors beyond assumptions, principles and concepts, to include methodologies, research questions and even ‘ways of seeing’. This is a change of focus from theories themselves to theories-in-use in research and through that to research practice more generally (see, for example, the final remarks in Prediger et al., 2009), where this practice can be seen as a theoretically guided activity. This move from connecting theories to connecting research, or research practices, or even to connecting researchers and the accompanying changing goals, suggests that there may be some value in revisiting the initial assumptions about the theoretical fragmentation and diversity of approaches in mathematics education and to ask what these mean in terms of research practices and researchers. Although we can sketch an a priori argument for the possible value of revisiting these assumptions, and that of exploration of the social conditions of research in mathematics education more generally, any particular implications, for understanding and managing diversity, must be demonstrated empirically.

In the study reported here I use techniques from social network analysis in order to examine evidence of fragmentation or diversity through existing collaborative research links between mathematics education researchers. We might expect that the theoretical and methodological divides would be seen in patterns of collaboration, since we would expect individuals using the same theories or working within the same perspectives to be more likely to collaborate with one another than with those working with different theories. This would then be evident in collaboration patterns with more densely connected groups loosely connected to other groups.

In looking at collaborations using social network analysis we create only one of many possible views of relations in mathematics education research, but an important one. Collaboration which results in a research publication implies a strong connection and investment of time. We might assume that even outside of the connecting theories endeavour, research collaborations are places where ideas meet, are elaborated and negotiated, both in the research process and in order to create a joint production which all parties are happy to put their name to. As such, evidence of connectedness across the field at the level of research collaborations would raise questions about assumptions of fragmentation of theoretical frameworks and research approaches, leading us to question the nature of this fragmentation.

This research is part of a larger project looking at the mathematics education research community in England and so focuses on collaborative links both within the
group of researchers based in England and between these researchers and the broader mathematics education community. My analysis suggests that these mathematics education researchers as a group are less fragmented in terms of their research collaborations than perceptions of theoretical and methodological fragmentation in the field, and in particular of mathematics research in England, might suggest.

**METHODOLOGY**

The study reported here using techniques from social network analysis to explore patterns of collaboration within the mathematics education research community using data collected from a large number of co-authored research papers.

The data used consisted of all research papers published in nine, largely English-language, international mathematics education journals between 2000 and 2009. Short editorials, book reviews, and announcements were excluded. Five journals were more general: *For the Learning of Mathematics*, *Educational Studies in Mathematics*, *Research in Mathematics Education*, the *Journal for Research in Mathematics Education* and *ZDM-International Journal on Mathematics Education*, along with four more specialised journals: *The International Journal for Technology in Mathematics Education*, the *International Journal of Computers for Mathematical Learning*, *Teaching Mathematics and its Applications* and the *Journal of Mathematics Teacher Education*. Journals in which English authors more commonly publish were preferentially selected, as the broader study, of which the reported research is a part, focuses on England. The final data set included 2264 papers in total, of which 1098 had more than one author.

In social network analysis (Hanneman & Riddle, 2005; Scott, 2000) individuals and the relations between them are modelled as the nodes and edges of graphs or networks. Tools from graph theory, along with some developed for use in the social sciences, are then employed to analyse and visualise these networks in order to explore or answer particular questions about their structure. In this study, researchers were represented by nodes and two researchers were linked by an edge if they had co-authored at least one paper in the data-set. Exploratory analysis of the resulting network focused on the number and sizes of connected sub-graphs (or components), how centralised the network was, whether there were relatively isolated sections, and how dense the connections were. Some of these measures are based on the distances between nodes within connected components of the network: the *degree of separation* of a node is its average distance from all other (connected) nodes and the *average degree of separation* for the component is calculated by taking the average of this value across all the nodes in the component. This type of measure can be used to make comparisons with other networks. *Cut-points* are nodes or edges which would disconnect sections of the network if they were removed. These can be used to explore how robust the features of the structure are, and as a measure of ‘connectedness’ by asking how hard it is to disconnect groups of nodes.
RESULTS & DISCUSSION

The list of 2264 research papers was used to generate a list of authors publishing in the journals and to identify who they had written papers with. Overall 2199 unique authors were identified, 1737 of whom had written a multiple authored paper in at least one of these journals within the ten year period specified. A further 462 authors only produced individually authored papers, i.e. they had not collaborated on any papers in the dataset. Most of the authors (66%) published only one paper within these nine journals, with 17% publishing two papers, 12% between three and five papers and only 3% of the authors publishing more than five.

Figure 1: Visualisation of the collaboration network of links between authors, generated using the freeware package Pajek (http://pajek.imfm.si/doku.php)

Figure 1 shows the visualised network of collaborations produced from this data. In the figure, researchers are represented by nodes and two researchers are joined by an edge if they have published a joint paper (authors who did not collaborate at all are excluded to save space but could be represented as 462 additional isolated points). The most striking feature of the network is that it has a single ‘giant’ connected component, seen here at the top left, containing 28% (or 612) of the authors. The remaining authors are all connected through collaboration to smaller groups of at most 27 authors. This pattern of one giant component and a number of much smaller connected components is characteristic of scientific collaboration networks (Newman, 2001, 2003) and is a pattern which proved to be robust to the addition and removal of particular journals (although the percentage coverage of the giant component varies).

I will use two particular results from my early descriptive analysis to address the question of the fragmentation of the field at the level of researchers. The first result
is that theoretical divides or other fragmenting factors discussed in the literature aren’t seen at the level of patterns of collaboration. Of course within small research groups this is the case, with many joint papers turning into a dense cluster in the network. However, looking for larger groups we see little evidence of clustering.

Analysis of the structure of the giant component to identify subgroups of more densely connected researchers reveals a ‘small worlds’ structure (Newman, 2001). This is as expected; small world structures arise naturally in many diverse systems. The average degree of separation of two nodes within the giant component is 7.6 i.e. researchers are connected to one another by on average a chain of only six or seven intermediate collaborators, with a maximum distance of 18. Overall the giant component is characterised by small distances between researchers and few (significant) cut points meaning that the structure is quite robust to the removal of ties or individuals.

This can be contrasted with patterns of collaboration in the hard sciences which are traditionally considered to be much less fragmented and more theoretically and methodologically coherent than mathematics education research. The measures of distance within the giant component are comparable (although his study used a much larger data set) with measures from Mark Newman’s study of collaboration patterns in the hard sciences (Newman, 2001). In fact, Newman’s study showed average distances of between 4 and 7 in six databases drawing on different areas of research and of 9.7 in a computer science database. In other words, there is a subset of mathematics education researchers with mutual patterns of collaboration which are not obviously more dispersed than those found among scientists in the hard sciences, particularly when compared to the sciences with lower average numbers of authors per paper. Unfortunately there are few similar studies of other social sciences with which to compare.

The second result from the network analysis relates to the proportion of the authors found within the giant component, in other words the proportion of authors connected to a significant number of other mathematics education researchers by collaborative links. Only 28% of all the authors publishing in the nine journals over the ten year period were connected in this way (or 35% of those who had published collaboratively). This is a low proportion compared with that found in studies of the hard sciences, which was over 50% in all subject areas examined and closer to 80 or 90% in many (Newman, 2001). The low proportion in the data is relatively robust to the removal, interchange or addition of journals (any 8 of the 9 producing a giant component covering 20-30% of the authors). This suggests that the low proportion is not just an artefact of the particular publications selected, nor a result of the relatively small number of publications being considered. Of the 1587 authors outside the giant component it is useful to separate them into two groups: around 70% of these have collaborated, but their collaboration still leaves them relatively isolated. Most are connected through collaboration (meaning that we consider their
collaborators and their collaborators’ collaborators and so on until there are no more connections to exploit) to only a small number of other researchers (on average to 4.4 others outside the giant component). The other 30% of authors outside the giant component have not collaborated on any papers within the data set. This significant proportion of authors who have not collaborated in the data set (21% of the total authors), along with the large number of papers in the original data with only one author (52%), is another point where the results differ sharply from patterns of collaboration familiar from the sciences. Even within subjects like mathematics, where working alone has traditionally been seen as the norm, there has been a trend of increased collaboration (Burton 1999 cited in Burton & Morgan, 2000). It would make an interesting further study to look at whether trends in mathematics education tend towards more or less collaborative research.

Interpreting this result requires more information and further research. We might ask what it means in practice to be inside or outside of the giant component in this maths education collaboration network. A researcher may tend to work alone but be actively involved with the community in other ways, or they may work alone as a result of geographic or institutional isolation; they may largely collaborate outside mathematics education, straddling several fields, or they may have published only one or two papers as part of a doctorate before leaving academia; they may publish occasionally but see the bulk of their work as lying outside research, in teaching, policy-making or administration for example. Clearly there are many different research profiles compatible with a position in the relatively unconnected sections of the network diagram, and so any single account will fail to capture this diversity of experience. Within the UK, education academics work in widely differing institutional contexts, with different patterns of research funding and different balances between their research and teaching functions (Lawn & Furlong, 2007; Oancea, 2005); additionally the career backgrounds of academics differ with many second-career researchers with varied experiences of research training (Mills et al., 2006). It may be that the giant component can be interpreted as representing a core of research-focused academics who focus on mathematics education and that the fragmentation of mathematics education can be partly understood through the relatively small size of this core with respect to the overall number of people publishing in the area.

FOCUS ON COLLABORATION IN CONNECTING THEORIES RESEARCH

Debate about the strengths and weaknesses and mutual compatibility of particular theories has a long history (Bikner-Ahsbahs & Prediger, 2006; Cobb, 2007; Sierpinska & Kilpatrick, 1998; Sriraman & English, 2005) and more recently this tradition of abstract discussion has been joined by moves (within the working group on theory at CERME 4, 5 and 6 and associated special issues of ZDM) to take a more practical approach to the problem by exploring strategies to connect theories in the context of empirical research.
One consequence of the move from debate about theories to practical strategies to connect theories is that it seems to have necessitated a different sort of engagement with the idea of theory. Discussions of the different ways in which theory can be defined for the purposes of connecting theories (Bikner-Ahsbahs & Prediger, 2006; Radford, 2008) or thought about in relation to empirical research (Cerulli, Trgalova, Maracci, Psycharis, & Georget, 2008; Prediger, 2008) can be seen as grappling with the difficulty in separating theory from theory-in-use. The use of ‘static’ definitions of theory has been challenged, introducing the idea that theories are tied up in the work of those who use and write about them and hence that a more fluid or ‘dynamic’ idea of theory is needed for a discussion of connecting theories through research (Prediger et al., 2008). In particular a possible change of emphasis has been suggested from networking theories to ‘the networking of research practices’ (Prediger et al., 2009, p. 1534, emphasis in original).

These changes of focus have brought with them the need to reconsider what might be the possible aims or goals for explorations of diversity and attempts to connect theories (ReMath first deliverable quoted in Artigue et al., 2009; Prediger et al., 2009). Across the papers of the CERME working group and the ReMath project there are discussions of important outcomes in terms of the ways researchers see their own work differently as a result of working with others who use different theoretical ideas (e.g. Trgalova, 2008). These experiences are described as a valuable part of the work by many, yet it seems clear from the discussion that it is an individually experienced result of the actual process of engaging with others in theory-focused research collaboration, rather than an insight which can be shared in a straightforward way with others or which might be seen as a tangible result of the research. Given the experiences reported, a valuable additional way of looking at the research of the CERME working group on theoretical and methodological diversity, and of thinking about the ways in which it could contribute to reducing the fragmentation of the field, might be to think of it as an exploration of the potential of collaboration, where that collaboration focuses on connecting theories and approaches to research, to act as a mechanism for change in dealing with fragmentation in mathematics education research. This move is consistent with the change of focus from connecting theories to connecting research practices.

CONCLUSION

I argue that the significance for mathematics education of the relative ‘closeness’ of researchers working with very different approaches and from different theoretical perspectives within the field of English-language publications is in the potential it suggests for using these collaborations to develop a greater sense of coherence in the field. Strategies developed from the connecting theories literature could play a role in developing the potential of existing collaborations. To explore this potential further suggests the need for work on the nature of existing collaborations and the extent to which researchers bring different theoretical perspectives into focus and debate when
collaborating, and whether this process could come to be seen as part of what is worth reporting about the research process. In doing this it would be important to recognise different ways of collaborating and working. The pattern of working illustrated in the data here suggests that in order to use collaboration as a mechanism to increase the coherence of mathematics education what may be needed is not necessarily more collaborations between researchers working within different theoretical traditions or with different approaches, but the exploitation of existing collaboration and the creation of new collaborative links to researchers without existing links.

The exploration of collaboration patterns in mathematics education research reported above raises some interesting questions about the assumption of fragmentation in the field based on theoretical or methodological divisions. Much fragmentation undoubtedly exists at the level of researchers collaborating, with high proportions of papers single-authored and a large proportion of the researchers publishing in mathematics education relatively isolated within the field in terms of collaborations. However there exists a smaller core of researchers who, despite differences of approach and theoretical perspective, remain quite closely connected through a relatively robust network of collaborative links.

The original study reported here was not designed to explore collaborative research connection across the whole mathematics education research community but instead to explore those of researchers working in England, and consequently it focused on research published in English. The result is that while we can draw conclusions about patterns of collaboration within the English-language literature, researchers who do not publish exclusively in English are systematically excluded or misrepresented, and so we need to take care in drawing conclusions about the whole field of mathematics education research. We might ask what degree of clustering a broader dataset would reveal, in particular around language groups. Education research generally differs from the sciences studied by Newman, in that English is not the only international publishing language although it remains dominant. Despite this limitation I would argue that the lack of evidence of strong disconnection among researchers’ collaborations warrants further exploration.

An issue in using network analysis and visualisations of collaborations in this research has been the ease with which one can move between the language of network analysis and the use of overlapping language employed generally within mathematics education research to discuss the state of the field in terms of methodological and theoretical diversity, and more specifically to the language of connecting theories and practices used by some in the CERME working group. The ideas of fragmentation and of connections or connectivity are particularly problematic here and care must be taken to trace differences of meaning between these terms as they are used in different contexts. Also the application of these terms in turn to theory, research, research practices, approaches and researchers can
unintentionally blur arguments and ideas which are specific to one into the others. A question raised here is what it might mean, and whether it is reasonable, to talk about the community of researchers in mathematics education as fragmented, and how this fragmentation might relate to theoretical or methodological fragmentation.

I see the use of network analysis techniques to explore collaboration patterns as just one of many possibilities for exploring the social context of research with a view to inform discussions about possibilities for reducing the fragmentation evident in mathematics education research. This could be an important complement to concrete attempts to connect theories and more abstract debates about the role of theory in research, the range of theories and the different approaches to research found in maths education and their implications for the knowledge production of the field. The social as well as epistemological, ontological or conceptual causes of theoretical fragmentation mean that even if we were to satisfy ourselves about the connectivity or otherwise of theories in mathematics education research, we would still find that fragmentation and diversity remained. The exploration of theories as theories (on an abstract level and through concrete attempts to connect theories and research), and the exploration of theoretical diversity as it has arisen from the social conditions of research, are both necessary in order to address the broader fragmentation of the field.

REFERENCES


