

THE POSSIBILITY OF A PERSPECTIVAL RESEARCH ON MATHEMATICS LEARNING

Tim Jay

Graduate School of Education, University of Bristol

The difficulty in reconciling the differing commitments – ontological, epistemological and methodological – of the various perspectives on research on mathematics learning is well established. As an example of this difficulty, there will be some discussion of cognitivist (e.g. Anderson, Reder & Simon, 1996, 1997) versus socioculturalist (e.g. Greeno, 1997) perspectives on mathematics learning. The claim will be made that problems of language, pace Wittgenstein (1953) and Derrida (1997), are at the heart of the apparent irreconcilability of these two and other perspectives, and that this apparent irreconcilability can be at least partly remedied through a post-structuralist, semiotic, approach. Some practical examples of the problem and some advantages of this approach are described and discussed.

PERSPECTIVES AND METAPHORS

My introduction, as many others', to some of the knotty theoretical problems faced by mathematics learning researchers, was through the papers of Anderson, Reder and Simon (1996, 1997) and Greeno (1997). Anderson et al. (1996) sets out a set of 4 purported claims of situated learning researchers; that "action is grounded in the concrete situation in which it occurs" (p.6), that "knowledge does not transfer between tasks" (p.6), that training by abstraction is of little use (p.8) and that instruction needs to be done in complex social environments (p.9). Each of these claims is dismissed in turn, the authors referring principally to cognitive psychology literature in their critique. The point of view expressed by Anderson et al. is that learning is fundamentally an individual process. They claim that whilst they unreservedly recognise the "profoundly social nature of the human species" (p.20), this social nature is best researched by analysing the "complex social situation into relations among a number of individuals and study the mind of each individual and how it contributes to the interaction" (p.21).

The response from Greeno (1997) consists in an attack, not on the evidence for the critique, but on the status of the four claims themselves. Greeno argues that the four claims set out by Anderson et al. demonstrate a misunderstanding of "the important differences between cognitive and situative perspectives" (p.5), and constitute something of a straw man for the cognitivists to attack. The substance of Greeno's response is that cognitivist and situative researchers differ primarily in terms of their 'primary focus of analysis'. The primary focus of analysis for the cognitivist is the set of processes and structures that exist within the individual mind, whilst the primary focus of analysis for the situativist is "at the level of interactive systems that

include individuals as participants, interacting with each other and with material and representational systems” (p.7).

Recently, two books have been published, Anderson (2007) and Sfard (2007), that set out updated theoretical frameworks from the two perspectives. There is still a very clear division between the two. Anderson (2007) involves the development of the ACT-R (Adaptive Control of Thought - Rational) architecture for creating models of human thinking, using data from human participants solving problems together with fMRI scans of brain activity in order to provide evidence for the validity of those models. The models created in ACT-R are capable of learning from instruction, but not spontaneous learning, or learning from peers. So whilst Anderson’s perspective has expanded in order to incorporate some neuroscientific thought and method, there has been no incursion into sociocultural thought. Sfard (2007) develops a theoretical framework centred on ‘commognition’, emphasising the fact that cognition is communication. However, the model is firmly sociocultural, and Sfard is explicit about the fact that she is concerned with thought that is exclusively human; that involves language. This approach precludes the researcher from incorporating aspects of cognitivist method into the study of learning.

THE LANGUAGE PROBLEM

This paper focuses on language in the reconciliation of perspectives. In fact, rather than treating this exercise as a reconciliation this paper, as is suggested in the title, takes the position that a more fruitful approach may be to abandon perspectives in favour of an aperspectival approach; binding the researcher to as few ontological and epistemological commitments as possible. This paper has already highlighted language as a key factor maintaining the dichotomy of perspectives. Researchers on both sides claim that their point of view has been misunderstood or misinterpreted by the other side. The problem to solve is that of dealing with a dichotomy that is obstructive and misleading. The solution we want to achieve is a theoretical framework that allows us to talk about individual cognition and learning, about sociocultural objects and processes and learning, and about interactions between these. Two approaches suggest themselves. The first is Derrida's (1997) deconstruction. Deconstruction feeds on dichotomy, subsuming the two sides of a dichotomy within a more comprehensive account. The second is Wittgenstein’s (1953) philosophy of language. Wittgenstein's later period involved a rejection of philosophical problems, and a claim that what appeared to be philosophical problems were in fact linguistic puzzles.

These two approaches, the deconstruction of the cognitivist/socioculturalist dichotomy, and the reconstitution of problems of ontology as problems of language, appear entirely compatible. In fact, it seems that to apply one approach is to apply the other. A demonstration that what appears to be a dichotomy is in fact an incompatibility of two language games appears to be an instantiation of

deconstruction. A deconstruction of a dichotomy appears to be equivalent to the founding of a language game in which that dichotomy dissolves. So we aim to solve the problem of this obstructive dichotomy then, by applying an analysis informed by Derrida and Wittgenstein. The primary resources for the framework are the constitution of the research process as a set of language games and the use of a semiotic account in order to construct the framework. Some previous literature has considered a possible role for this form of analysis in mathematics learning research, although the focus has generally been on the use of post-structuralist, semiotic analysis as a means of describing classroom activity (e.g. Brown, 1997; Evans & Tsatsaroni, 1996); and as such these have taken a sociocultural perspective on mathematics learning. In this paper, we are working at a different level, addressing questions of mathematics learning more generally.

THE PROBLEM: A RESEARCH EXAMPLE

This section presents an example of research in mathematics learning, addresses the issue of what exactly is the problem with the existing dichotomy of perspectives, and introduces questions that are difficult to engage with within this dichotomy.

Jay (2009) makes use of priming protocols, employed within experimental method, to demonstrate a relationship between number knowledge and strategy use in young children. The study focused on children's use of the 'tie' strategy, for solving near-double single-digit addition problems with solutions greater than 10. For example, the problem '7+8' could be solved by solving '7+7+1', making use of the 'doubling fact' '7+7=14'. In this study, children between 7 and 9 years of age took part in two activities; a) solving a set of single-digit problems with solutions greater than 10, reporting the strategy used following each problem (children had a free choice regarding strategy), and b) completing a set of priming trials designed to test for the automaticity of activation of doubles in response to single-digit stimuli (testing whether '7' activates '14', for example). The priming trials were based on previous research demonstrating automatic processing of numerals and relations amongst numerical information by, for example, Garcia-Orza, Damas-Lopez, Matas and Rodriguez (2009) and Reynvoet and Brysbaert (2004). The sample of children was divided into two groups, one group consisting of all of the children who used the tie strategy at least once whilst working through the set of addition problems, and a second group consisting of all those who did not. The two groups were then compared with reference to data from the priming trials. This analysis showed that only the children using the tie strategy showed evidence of automatic activation of doubles. This in turn suggests that automatic activation of doubles is a key resource in children's development of the tie strategy.

Up to this point in the description, Jay (2009) will appear to be situated firmly within the cognitivist perspective, utilising experimental psychology methods. However, this study is best seen as situated within the primary school classrooms in which the

research took place. Conversations with the children forming the sample for the study (5 classes in 2 schools took part), and with their teachers and the mathematics coordinators for the schools, made clear a number of points with a bearing on the interpretation of the findings. Firstly, all of the children involved in the study knew about doubles, and were able to give the double of a single-digit number with only a short delay. Secondly, classroom mathematics instruction had involved making children aware of the variety of strategies for solving simple addition problems, and encouraging children to make use of these strategies in order to increase the efficiency of children's problem solving, since at least Year 1 (the children in the study were in Years 3-5). Mathematics lessons took place for one hour every day for all children in both schools.

Within the wider context of the classes and the schools in which the children's mathematics education is taking place, the results of Jay (2009) raise some interesting questions. Firstly, why do some children have automatic activation while others do not? A second question is; what can we do in order to help children develop and use the tie strategy and other efficient strategies for solving arithmetic problems? This is not just a developmental issue; some children in each year group from year 3 to 5 (age 7-9) did not automatically activate doubles in response to single-digit stimuli. It is not just a matter of having had experience of doubling; all children could calculate a double without difficulty and all had significant experience of having been taught about doubles and their relevance for calculation in the classroom. Now, the question I would like to pose next is: within which perspective ought we to proceed? The problem faced at this point is that neither the cognitivist nor the socioculturalist perspective offers an appropriate language for asking the kinds of questions we are going to want to ask. This research situation is a clear example of one in which we will need to ask not about individual thought and learning processes, not about social or group cognition, and not about broader sociocultural process, at least not in isolation from one another. What is needed in order to really address the problem are questions that focus on the *interactions* between these processes. As long as the cognitivist and socioculturalist perspectives are considered separate and mutually exclusive, it is very difficult to ask question that address interactions between biology, cognition, classroom interactions and wider sociocultural objects and processes.

A POST-STRUCTURALIST ANALYSIS

The purpose of this treatment is to reject the dichotomy between accounts of the individual and accounts of the group. This leads us towards a distributed, situated account of thought and learning. It also leads us to deconstruction. A first principle of this approach is to say that all we can talk about are signifiers and relationships amongst signifiers. All thought consists of signifiers and relationships amongst signifiers. This is very much related to Derrida's suggestion that there is 'nothing outside the text' (e.g. Derrida, 1997) – there is nothing we can say about thought that

is outside what we can say about signifiers. If we want to talk about learning, then we consider all aspects of learning part of the one single-order text, whether biological, cognitive, sociocultural or otherwise.

So, what do we have to do in order to demonstrate that a semiotic account functions as a framework for theoretical discussion of learning? What affordances must such a framework have? I want to claim that there are just two criteria for a theoretical framework to meet; we want to be able to demonstrate the fact that we can ask the questions that we want to ask about learning and we want to be able to demonstrate the fact that those questions are answerable. Of course, we will have to be able to say what is meant by conventional uses of terms such as 'knowledge', 'concepts', 'learning', 'mind', or at least construct meaningful and useful definitions of these terms within the framework, but that is just one example of a criterion that is necessary for the two principle criteria to be met.

The purpose for rejecting the dichotomy between individual and social accounts of learning is that it is becoming increasingly clear that any genuine, meaningful, account of learning is going to depend on being able to describe the interplay between individual and social factors. So, the kinds of questions that we want our theoretical framework to help us ask and answer include all those asked by cognitivists (lots of 'what' questions), all those asked by socioculturalists (lots of 'why' questions) and those addressing the interaction between the two.

APPLICATION TO EARLY NUMBER

How is the semiotic framework going to work? What is it going to look like? This section presents an example involving a child's developing understanding of number, taking the number 4 as an example. A child, during the course of their first few years of life, will come to recognise and use several signifiers related to the number 4. Some examples: the Arabic digit '4'; the written English word 'four'; the spoken English word 'four'; the four fingers (without the thumb) of one hand; and the arrangement of 4 dots on one side of a 6-sided die. There are likely to be many other signifiers directly related to the number 4. For example, my son, at the time of writing is three years old. He identifies very strongly with the number 3. If he sees the digit '3', he will often say something like, "I'm that number". I imagine he will continue to say something like that when he is four years old, although I expect it will be interesting to observe how this kind of statement will develop as he comes to adjust his differentiation of his use of number-signifiers for their various purposes.

Now, we have said above that these are examples of signifiers related to the number 4. We might want to ask, what then is the signified? Is it '4'? This is a very important step in the development of our account. We cannot talk about a signified, only about other signifiers and relationships amongst signifiers. If we can't talk about signifieds then how do we deal with questions of meaning? We might normally want to talk about the meaning of a particular signifier associated with a signified. It might still

be useful to think in this way, but we also need to remember that we have no way, within the language system available to us, to talk about signifieds. So we try to describe signifieds – by instantiating new signifiers and new relationships amongst signifiers, in an attempt to, little by little, close the gap between signifier and signified. The gap can never be completely closed, because we are using a language system to describe and define something outside of that language system. In Lacan's (e.g. 2001) framework, we are trying to bridge the gap between the Symbolic order and the order of the Real, the unbridgeable gap referred to as the 'lack'.

So what is the meaning of the Arabic symbol '4'? Its meaning consists in the relationships that it has with other signifiers. '4' has a relationship with the other signifiers given in the list above. It has a relationship with the signifier for 'a sense of four-ness'. It has relationships with signifiers of '3' and '5', because we are habituated to seeing and thinking about '4' in its place in the sequence of natural numbers, or integers. An important point must be made here regarding this account of meaning. The statements above could be read in a relatively simplistic way, if one assumed that relations amongst signifiers are constrained by the bounds of an individual brain or mind. However, once we reject this boundary it is clear that the simplistic reading needs further development. Here again there is correspondence with Wittgenstein's (1953) account of language. 'Meaning is use' and the Private Language argument tell us that no analysis of meaning can take place entirely within the bounds of an individual brain/mind. Meaning of language (consisting of signifiers) consists in the role that language (that set of signifiers) plays in acts of communication amongst brains and minds. So, the analysis of the meaning of '4' can be developed as follows: The meaning of the signifier '4' consists in the role that '4' plays in acts of communication both intra- and inter-personally. This is close to the position taken by Sfard (2008), although Sfard appears to take a position restricting analysis of cognition to exclusively human modes of thought. It is clear from the cognition literature that the deliberative portion of human thought is largely dependent on automatic, uncontrolled processes (see for example, reviews of the Stroop effect [MacLeod, 1991], of priming studies [e.g. Kinoshita & Lupker, 2003]). Our response to the signifier '4' is largely involuntary, at least in the first few milliseconds after perceiving it. However, involuntary or not, there is no reason for doubting that the meaning of a signifier consists in its role in an act of communication – after all, involuntary responses to a signifier are not limited to intra-personal communication. Involuntary responses are also clearly apparent in inter-personal communication (see again priming studies). This is not to say that these data regarding automatic responses contradict Sfard's theory, but rather to say that Sfard's theory does not *encompass* these aspects of what we know to be a fundamental aspect of thinking and learning.

Defining key terms

We are in a position now where we need to define some key terms. If we want to be able to talk individual and social learning, subsuming cognitivist and socioculturalist perspectives, then we are going to need to know what we want to mean by terms such as 'concept', 'knowledge', 'learning', 'understanding', 'mind', and so on. These terms themselves have potential to act as barriers to interdisciplinarity, due to substantial differences in their definition and interpretation by researchers operating from different perspectives. Each term is a reification, brought into being by the researcher/observer. A 'concept' is nothing but an observer's definition. So the substantially different definitions of 'concept' by, for example, cognitivist and socioculturalist researchers, constitute a significant barrier to communication. It is important that such terms ('concept', 'knowledge', 'understanding' and so on) are recognised as reifications, in order to provide a means and an arena for the negotiation of their meaning. Some initial sketches of meaning might be made as follows:

Signifier: A signifier is a unit of meaning. It might be a word, symbol, image or object. It might have a physical instantiation (ink on paper, physical object, photograph, audible sound and so on). The meaning of a signifier consists in its relation to other signifiers.

Concept: The network of signifiers and relationships among them, that relate to a given signifier. So the concept 'addition' consists in the set of signifiers that relate to the signifier 'addition', plus the relationships amongst them. Such related signifiers might include words like 'sum', 'add', 'plus', the symbol '+' and addition facts such as '2+2=4'. Also strategies or algorithms for solving particular addition problems (we might, as researchers/observers, refer to these as 'count-on', 'min' and so on – others will not use these names, however the algorithm itself constitutes a signifier). It is clear that any concept is a dynamic entity. Concepts change as the focal signifier is associated with new signifiers, or relationships amongst a set of signifiers alter.

Mind: The set of signifiers, and relationships amongst those signifiers, for some definable set of individuals. We can talk about the mind of an individual – that is the set of signifiers for that individual (the external physical objects that have meaning for that individual, plus the neurologically instantiated signifiers for that individual, plus the relationships amongst these signifiers).

Understanding: This seems to be a term that refers to the intersubjective aspects of 'concept'. One might be said to have understood a concept when one has developed a signification network that is sufficiently similar to that of a community with which one wants to engage with reference to that concept.

Two important points should be made apparent at this time. One is that it is clearly possible to define these and other terms within the proposed semiotic framework. That is to say that the framework appears sufficient for the discussions that we want

to have about learning. The second point is that the framework makes clear the fact that each term is a reification that, if it is to be used in any meaningful way, must be defined within a shared framework. The proposed semiotic framework appears to offer a sufficiently neutral arena for negotiation of such definitions amongst researchers from traditionally disparate or dichotomous perspectives.

RETURN TO THE RESEARCH PROBLEM

As a conclusion to this paper, we return to the discussion above regarding Jay (2009). What can be done to address the research problem; why do some children not automatically activate doubles, and what can be done to help them? What kind of research questions arise from the kind of analysis described above? At the heart of the proposed framework is an imperative to focus on the interactions amongst cognitive and sociocultural objects and processes. To address these interactions, we describe the situation in terms of signifiers and relationships amongst signifiers. Let us begin with a description of the research situation within the individual child. Firstly, we are interested in why it is that some children automatically activate doubles in response to single-digit numbers. This is to say that we are interested in why some children exhibit a very strong connection between the signifier '6' and the signifier '12', between '7' and '14' and so on. We are also interested in the role that this set of activations plays in the use of the tie strategy.

On the sociocultural side we are again interested in how number signifiers and relationships amongst them are used, but this time we are interested at the level of interaction amongst actors in the classroom. Stahl's (2005) 'group cognition' might be a useful way to think about some of the processes that might be involved in this activity, as long as we remember that within social processes we are dealing with exactly the same kinds of signifiers and relationships amongst signifiers that we deal with within the individual.

The first thing to realise is that this approach allows now to hold some apparently contradictory things to be true. So within any given individual in the classroom, the meaning of '7' consists in the relationships that '7' has to other signifiers. So one particular child might relate '7' to 'seven', 'holding up 7 fingers', 'my age', '6' and '8' (due to proximity on the number line). Other children might relate it to 'prime', some others to '14' (double 7). So, from the cognitivist perspective, we talk about *an* individual child that doesn't exist; we talk about a generalised, average, child. In fact, just looking at the meaning of '7', it is likely that, firstly, different children will relate different signifiers to '7', and secondly, the strengths of these relationships will be different. So '7' has a different (even if only subtly) meaning for each person in a classroom. Aside from all of the individually constituted meanings, there is also a socially negotiated meaning of '7' for the *classroom*. This is formed through interactions amongst members of the group. The negotiation of meaning of signifiers in the classroom is dynamic; there will be a constant dynamic interaction between

socially constituted meaning of signifiers and meanings of those same signifiers held by individuals. In turn, meaning for the classroom, is tempered by, and interacts with, meanings of wider and external communities and groups. So, for example, each child brings with them aspects of meaning from their own families. In the context of the research situation, this opens us up to dealing with some new questions, focusing on interactions between meanings constituted by individuals, and meanings constituted, or negotiated, by the group. In general, we can ask questions about how children's knowledge, accessible to cognitive and neuroscientific methods, interacts with (influences and is influenced by) classroom activity. Specifically with regard to Jay (2009), it would be informative to ask what differences there are in terms of engagement in classroom interaction between those children who exhibit a priming effect for doubles and those who do not. It might also be possible to investigate how knowledge such as this spreads through a classroom, through group interaction.

CONCLUSION

This paper has argued for a post-structural, semiotic, treatment of research on mathematics learning in order to reconcile traditional perspectives and allow the possibility of asking questions about interactions between individual/cognitive and group/sociocultural, aspects of learning. These kinds of questions offer an opportunity to engage with research on learning with greater depth than is possible with more restrictive, traditional, perspectives on mathematics learning research.

There is much more to do in the development of this approach. Existing research conducted from multiple perspectives can be interpreted within this framework in order to understand points of conflict and potential interactions more clearly. It will be important to thoroughly test the claim made here that this framework offers the potential to fairly represent existing perspectives on mathematics learning.

The post-structuralist approach provides a means of being explicit about what aspects of what questions of learning we are interested in, without excluding other aspects from the arena. We can be free to talk about what meanings are common amongst a particular population of children in a particular domain of mathematics, including trajectories of individual development, relationships between knowledge and understanding, knowing-that and knowing-how, as well as talking about individual differences within that population. We can also talk about classroom activity and the development of socially negotiated meaning through classroom interaction. As a result of using the same language, that of signifiers and relations amongst signifiers, in discussions of both individual and social thought and learning, some of the barriers to discussions of interactions between the individual and social are removed.

REFERENCES

Anderson, J., Reder, L. and Simon, H. (1996). Situated learning and education. *Educational Researcher*, 25 (4), 5-11.

- Anderson, J., Reder, L. and Simon, H. (1997). Situative versus cognitive perspectives: form versus substance. *Educational Researcher*, 26 (1), 18-21.
- Anderson, J. (2007). *How can the human mind exist in the physical universe?* New York: Oxford University Press.
- Brown, T. (1997). *Mathematics Education and Language: Interpreting Hermeneutics and Post-Structuralism*. Dordrecht, The Netherlands: Kluwer Academic Publishers.
- Derrida, J. (1997). *Of Grammatology (corrected edition) trans. Spivak, G.* Baltimore, Maryland: Johns Hopkins University Press.
- Evans, J. & Tsatsaroni, A. (1996). Linking the cognitive and the affective in educational research: cognitivist, psychoanalytic and post-structuralist models. *British Educational Research Journal*, 22 (3), 347-358.
- Garcia-Orza, J., Damas-Lopez, J., Matas, A. & Rodriguez, J.M. (2009). "2 x 3" primes naming "6": Evidence from masked priming. *Attention, Perception & Psychophysics*, 71 (3), 471-480.
- Greeno, J. (1997). On claims that answer the wrong questions. *Educational Researcher*, 26 (1), 5-17.
- Jay, T. (2009). The relationship between number knowledge and strategy choice: what we can learn from the priming paradigm. In Joubert, M. (Ed.) *Proceedings of the British Society for Research into Learning Mathematics*, 29 (3), November 2009, Loughborough University.
- Kinoshita, S. & Lupker, S. (Eds.) (2003). *Masked Priming: The State of the Art*. Hove, East Sussex: Psychology Press.
- Lacan, J. (2001). *Écrits: a selection, trans. Alan Sheridan*. London: Routledge.
- Neely, J. H. (1991). Semantic priming effects in visual word recognition: A selective review of current findings and theories. In Besner, D. & Humphreys, G. (Eds.). *Basic processes in reading: Visual word recognition*. Hillsdale, New Jersey: Lawrence Erlbaum.
- Reynvoet B. & Brysbaert, M. (2004). Cross-notation number priming at different stimulus onset asynchronies in parity and naming tasks. *Experimental Psychology*, 51, pp. 81-90.
- Sfard, A. (2007). *Commognition: Thinking as communicating, the case of mathematics*. New York: Cambridge University Press
- Stahl, G. (2005). Group cognition in computer-assisted collaborative learning. *Journal of Computer Assisted Learning*, 21 (2), pp. 79-90.
- Wittgenstein, L. (1953). *Philosophical Investigations*. Oxford: Blackwell.