HOW A GENERAL EPISTEMIC NEED LEADS TO A NEED FOR A NEW CONSTRUCT

A CASE OF NETWORKING TWO THEORETICAL APPROACHES

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Abstract: Two students were asked to carry out an activity about continued fractions in the presence of an interviewer. We present two separate analyses of a central episode from two theoretical perspectives focussing on what drives the students’ progress of coming to know. Making intensive use of the notion of general epistemic need, we show how such needs arise and are expressed, and how they lead to the need for a specific new construct. The paper concludes with reflections on the networking process, highlighting conditions that have supported the process and hence emergence of a more integrated view of the episode.

INTRODUCTION

Networking of theories has recently become an important endeavour of mathematics education (Bikner-Ahsbahs et al., 2010). We present a facet of an ongoing research project that investigates effective knowledge construction in interest-dense situations\textsuperscript{1}. We found it useful to proceed by networking two theoretical approaches, the perspective of abstraction in context (AiC) and that of interest-dense situations (IDS) with particular focus on two theoretical concepts: the need for a new construct, and interest. In both perspectives epistemic processes play a central role.

AiC focuses on the construction of knowledge by individuals or small groups, using a model based on three epistemic actions: recognizing constructs known from previous activity, building-with these recognized constructs, for example to solve a problem, and constructing new knowledge while building-with recognized previous constructs. While the model serves to analyse the central stage of the emergence of a new (to the learners) construct, the researchers postulated that a necessary condition for such emergence is that the learners experience a need for the new construct (see Schwarz, Dreyfus, & Hershkowitz, 2009, for a detailed description).

Interest-dense situations (IDS) focus on the epistemic processes within social interactions. These are situations that are constituted by students who are collectively deeply involved in a mathematical problem, constructing mathematical knowledge in a deep way and highly valuing the mathematics they consider. This approach uses an epistemic actions model that comprises three collective epistemic actions: gathering and connecting mathematical meanings and seeing mathematical structures. Any IDS lead to structure seeing, and students who are engaged in an IDS show situational interest. Situational interest holds when an individual experiences involvement and meaningfulness of a topic (see Bikner-Ahsbahs, 2005, for a detailed description).
In a previous paper (Kidron, Bikner-Ahsbahs, Cramer, Dreyfus, & Gilboa, 2010), we exhibited the methodology of networking the two approaches on the basis of common analyses of interview protocols from both perspectives. One benefit of networking was the insight that besides interest (IDS) and the need for a new construct (AiC), a more general epistemic need (GEN) can be the driving force that makes students to progress in learning processes according to the challenge they meet within a situation, individually and socially. The GEN is the need to develop an initially vague idea further, leading to a more definite one, in accord with Davydov’s (1972/1990) view of abstraction. The GEN is experienced according to the demands of the situation and hence becomes specific, for example as a need to be more precise that is shown by student actions and can be observed.

In the present paper, we continue the previous research using a different set of data. We carried out a networked analysis of an epistemic situation from the two perspectives concerning the roles of the GEN for constructing knowledge. The general result of the analysis is a holistic view of the learning process integrating both perspectives. An important specific result is that the GEN develops, at least in this case, into a need for a specific new construct as postulated by AiC.

**SETTING AND TASK**

Two grade 10 students work on a task about the continued fraction \(1 + \frac{2}{1 + \frac{2}{1 + \ldots}}\) in an interview situation. The main role of the interviewer is to support these students with hints (beginning with very weak hints) when they fail to progress. The students were asked to calculate the first seven fractions, expressing them as simple fractions; it was suggested to denote the \(x^{th}\) fraction by \(f(x)\), beginning with \(f(0)=1\). The students were then asked to represent the first 20 fractions as decimal numbers, to look for patterns, make conjectures, and explain why these conjectures are true.

The students discovered that the decimal numbers in even places (\(x\) even) begin with a 1, the decimal point being followed by a number of nines, and the ones in odd places begin with a 2, the decimal point being followed by a number of zeros. They also noticed that the number of nines, respectively zeros grows as more fractions are computed.

The students later observed the following rule: when the \(x\)-value passes a perfect square, the numbers of nines (zeros) increases by 1 and becomes equal to the root of that perfect square. When formulating this rule, they were talking in terms of a “space of places” (places referring to decimal places). They also noticed that this rule was valid only approximately. The rule may appear as an expression of a polynomial growth law to the expert, but at least at this stage, the students considered it only locally, for fixed \(x\).
THE ROLE OF THE GEN FOR ABSTRACTION IN CONTEXT

The GEN and the seeds for a constructing action

In the transcript (available from the researchers) we observe some phenomenological identification of patterns (in utterances 719-1353), which we may consider as seeds for constructing actions that take place later. By phenomenological, we refer to the fact that the elements of the sequence are considered as strings of digits rather than as numbers. The expression “space of places” (which we will abbreviate henceforth as SP) appears in 818 and there is some indication for an epistemic need in the efforts of the students to clarify this notion and to assign it a name. We submit that when somebody expresses the same phenomenon in different ways in an attempt to associate meaning with it, we have an indication for a GEN. This was the case in relation to the expression “space of places”. In addition to the different ways the students expressed the SP, we also discern a double interpretation of the SP: (i) SP as an interval on the $x$-axis, which numbers the elements, specifically the interval in which the number of nines/zeros remains the same; and (ii) SP as a part of the decimal expansion of $f(x)$, specifically the part containing the nines (or zeros). This double interpretation reinforces our interpretation of the SP as a seed for later constructing, as something that is not precise and needs to be elaborated.

The Role of the GEN in constructing actions

We first relate to one specific aspect of the GEN, namely the need to understand the present situation in terms of the previous knowledge or previous experience, to engage with the challenges offered by the task. Then, we note how this need and the limitations of the previous knowledge [specifically that the previous knowledge was adequate to empirical computations while the strings of digits were explicitly written and observed] lead to other specific aspects of the GEN, namely the need to be more general and the need to clarify. Finally, we will observe how these specific aspects of the GEN lead to the emergence of the need for a new construct as postulated by AiC.

After the phase described above as phenomenological, we observe a striking change in the students’ attitude and way of thinking: They start giving reasons rather than only phenomenological descriptions. We interpret this as a consequence of the interviewer’s initiative to ask questions concerning the SP for $f(1’000)$ (in 1354) and for $f(1’000’000)$ (in 1397). The students express their need to understand the new situation in terms of their previous empirical experience, and express their thinking that they need to do all 1000 computations by recursion (in 1359-1362). The limitation that results (“we cannot do all the 1000 computations”) leads to the need to be more general and to apply their “theory” that the length of the SP approximately equals the square root of $x$ more generally (1382).

The next initiative of the interviewer, the question “How would it work then?” (in 1424) - again causes the students to experience a limitation of their previous experience and it leads them to the need to think in a more general way. This need directed the students towards the beginning of a constructing phase in which infinity
plays a role. We note the important role of the interviewer but at the same time also the fact that the notion of infinity was expressed on the students’ initiative. The students express a need to understand the meaning of “infinite[ly many] zeros” (in 1429), and ”infinite[ly many] nines” (in 1431) as well as the meaning of a sequence approaching a number: “it keeps on leaning closer to zero-, closer to two, both numbers” (1427). Hence, we observe constructing actions that relate to convergence and accompany the students’ extension of the growth pattern of the SP from the initial 20 elements to 1’000, 1’000’000 and beyond, to what the students call infinity.

A first constructing action that we denote $C_0$, *Convergence as coming very close to* ..., appears in 1418 and more clearly in 1427. Then, we observe $C_1$, *the Potential Infinite* process view in 1427 (see above) but also, for example, “If one looks at it precisely, it never reaches two, even if there are infinite nines, after it there always comes seven three two whatever, can be anything the following numbers, we have not even looked at them yet, could be that they have a pattern too, but I don’t, personally I don’t see anything there (M laughs)” (in 1473). $C_1$ is accompanied by $C_2$, *Infinite as a façon de parler: very, very large but finite*, as well as $C_3$, *the strange infinite object is legitimate only in the mathematical world*, for example in 1356:

1354. T: Because one, one nine ninth is namely one point nine nine nine nine nine nine nine nine, a-nd two, because one plus nine ninth is definitely two but nine, one ninth, is zero point one one one one one

1355. M: Yes but then, if you want to make nine ninth, then it would be two

1356. T: Theoretically (M laughs)

$C_3$ is similarly expressed in “If, if you insert infinity it theoretically equals two” (in 1437), which somewhat later leads them to write “f(∞)=2” on their worksheet. Another aspect of $C_3$ is the transition from infinite as a façon de parler to infinite as a legitimate object in the mathematical world.

$C_1$, $C_2$, and $C_3$ develop in parallel: At the same time that the potential infinite process view is developed, the students also begin manipulating the infinite as a legitimate mathematical object as they have done previously for large but finite numbers. The occurrence of so different (and somewhat contradictory) constructing actions in parallel places heavy demands on the students. This leads to a feeling of unease, of confusion, which is expressed in 1473 (cited above) and causes the expression of a need for a new construct in 1478: “The best would be of course if we had a functional equation right? Thus if one could say exactly, f of x equals (...)” We can see this need as a consequence of the limitations of the students’ previous experience: In the present situation, they are not able any more to use what they know from the finite case. Therefore, a need for a new view is expressed. This need leads to the construction $C_4$, *transition from a numerical way of thinking (with empirical results calculated by the students) to a more general way of thinking (which does not depend on specific cases)*. A new construct is needed to permit this transition. The need for this new construct is explicitly expressed in 1478. During this search, the students
continue giving reasons rather than only describing phenomena. This new approach in which the students explain their way of thinking provides evidence for the passage from the seeds of constructing to the beginning of the constructing process. The seeds of constructing also influenced later constructions. This appears, for example, when the students express the distance of $f(x)$ from 2 by means of their idea of SP (in 1534) and point out that the SP is the base 10 logarithm of the distance.

THE ROLE OF GEN IN INTEREST-DENSE SITUATIONS

The social interactions in IDSs can be regarded from a mathematical point of view as a flow of mathematical ideas that produces mathematical knowledge in an effective way by the epistemic actions of gathering, and connecting mathematical meanings, which lead to structure seeing. Figure 1 shows pictograms of the six phases of the analyzed episode. All are initiated by the interviewer (see the arrows). Phase I mainly consists of gathering, in phases II, III and VI gathering and connecting are merged. In phases IV and V, the students reach structure seeing including validating structures.

![Phase diagram of the analyzed episode](image)

Figure 1: Phase diagram of the analyzed episode (1333-1512), phase I: 1333-1353, II: 1354-1401, III: 1402-1423, IV: 1424-1454, V: 1455-1466, VI: 1468-1512

Epistemic actions serve to describe the flow of ideas and to investigate what drives the epistemic process in the flow and where it leads. A flow of ideas is a horizontal scanning of the mathematical aspects of a problem expressed in the utterances towards oneself and the other in order to describe, concretize, understand, progress, …, but also to inform the other, to take up her idea and develop it further, negotiate, explain, … It is an evolution of ideas associated with a given mathematical problem, building on previous experiences. Within a constructing process driven by a GEN the flow of ideas may lead to: recognizing an idea as fruitful, which may lead to further developing it; connecting the aspects together, which may lead to building-with a comprehensive view; seeing a structure or constructing something new; checking, understanding, making concrete and justifying the structure or the construct.

For example in phase III (1397-1423), the flow of ideas refers to how the digits of $f(x)$ for $x=1'000'000$ may look like. The status of the power law for the length of SP had been made clear before as a conjecture that leads to estimations about it. The focus now is on how the decimal numbers look as compared to 2. $f(1'000'000)$ cannot be computed explicitly. Therefore, the question how $f(x)$ looks for $x=1'000'000$, turns the argumentation into a hypothetical direction, connecting it to the SP and to how the sequence might go on:
But what we do know in any case is that there is a one before the decimal point, well not for one thousand and one, for thou- for one-
No, for one thousand and one there is a one in front of the decimal point, well no wait yes a two
That’s an odd number, yes
Two point zero zero zero zero zero
Yes because its an odd eeh place
Yes, so its very close to two already
Yes
Those are then about a hundred zero or so (laughs), and then comes some different number

In the end the last idea is confirmed (1422, 1423). This flow of ideas describes horizontally how the decimal number \( f(1'000'000) \) might look without going a step further. However, this horizontal scanning process implicitly produces the aspect of approximation to 2 that is recognized as fruitful for answering the interviewer’s question “and how would it work then?” (1424). It is further developed in the next flow of ideas as a process leading to structure seeing: “it keeps on going” (1425) (seed for infinite as a process), “an infinite number” (1426) (length of the decimal number), “leaning closer to zero, closer to two, both numbers” (1427), (structure seeing because of the leaning-key-idea of approximation, grasping the convergence to 2, and referring to both numbers, meaning converging from both sides), “but no never becomes 2” (sequential process of potential infinity), “there are always infinite zeros” (1429) (here the value of the digits connected to the process directs the view to the actual infinite), “it’s infinite that’s just it” (1430) (the length of the decimal number), “at the end there are infinite zeros or infinite nines, and there is something” (1432) (the image of the infinite length is rooted in the experience of the finite).

Here again an aspect is further developed that has been prepared by enlarging the size of x-values from 1’000 to 1’000’000. Before, the flow of ideas was concerned with the infinite length of the decimal numbers, whereas now the students take infinite as an actual value of \( x \), which they consider substituting in a functional term: actual infinity is reached (1434), structure seeing took place. This kind of substitution changes the view from function values to the variable \( x \). Based on that, the students several times use an if-then consideration and a flow of ideas about the conclusion arises: if ”we insert infinity” (1435), “will always be the same” (1436), “if you insert infinity, it theoretically equals two” (1437), “then it would be two” (1438). “One point nine period” (1440), “equals two then” (1442), “equals about two” 1443, “equals two” (1444), “so close, ah ok” (1445).

The students agree about the if-then-argumentation and the data but not about the conclusion. Here we have a flow of ideas leading to dispersing views. T is bound to the view of potential infinity (1437, 1443, 1445) whereas M reaches actual infinity
(“we can insert infinity” 1434, see also 1442, 1444). The difference between these views is not overcome because of their incompatibility. This is an experience of limitation and brings about a need for certitude referring to and repeating what the teacher has said. M recognizes an argument of their teacher as fruitful and together they reconstruct it: manipulations leading to 1.9999... = 2 as learnt in school.

In addition, the flow of ideas can lead to the experience of limitations such as not being able to continue solving the problem. This can happen when the students do not have access to tools that would help them progress, but it can also happen when they have a different understanding of an aspect that cannot be overcome in this moment. However, limitations do not necessarily have to lead students to give up; they can be overcome in different directions. They may lead to changing the conditions, going back to a clearer situation, taking a more general view; they may also lead to the need for a new construct (NNC). We now discuss some of these possibilities.

1. **Giving up**

   In this case the students are not driven by a GEN nor do they find an adequate expression of it (no GEN); for example in phase I they are ready to give up since the flow of ideas dries out because they have experienced that the power law is not always valid: “we could probably work on this all day long” (1340) and “yes but I think it’s enough” (1341).

2. **Changing the conditions**

   Changing the conditions with a potential for progress involves a mixture of a need for clarification and a need to progress: The demand to use the function value of 1000 that seems too big to compute, and the attempt to use the power law that cannot be applied directly, make them change the conditions. Instead of 1000 they take 100 first and then 1024, since these are numbers with well known square roots (1370).

3. **Going back to a clear situation**

   This reaction is concerned with a need for certitude: the demand to look at 1’000’000 makes them change the conditions first and then a need for certitude leads them to say what “but what we do know in any case is that eeh there is a one before the decimal point, well not for one thousand and one, for thou- for one-“ (1413), which is confirmed by T in 1414, and again in 1477.

4. **Taking a more general view**

   This may be connected to a need to be more general: the demand of the interviewer to look at 1000 or 1001 led to a limitation “one can’t say anything else” (in 1359). This caused a need for being more general “wait, we have our theory” (1360) that they applied afterwards.

5. **Expressing the need for a new construct**

   This need arises when the students describe what they would need in order to continue and why they do not have it yet: In 1473, T experiences a deep personal
limitation, “personally, I don’t see anything there” (see above for the full citation) although there is a specific epistemic need to be general. T tries to transfer his images of finite numbers to the infinite ones but this is not observable. Therefore he experiences a lack of tools to continue and that confuses him. The GEN is expressed by a need for a new construct in 1478 (see citation above).

**Situational interest empowers the students to act epistemically**

In 1467, the students turn to the task of justifying their conjecture, which makes them laugh. As before, the students do not take the demand to justify seriously. However, they value this task now as being more difficult. In 1469, the interviewer acknowledges “I find your last aspect just now most interesting”. This changes the situation completely. M confirms: “Yes that’s really is interesting how“ (1470), without laughing. T describes what is really interesting: “yes, so theoretically it keeps on leaning closer to two” (1471). Both students take the aspect of approximation as what is most interesting. The number 2 is understood as the leaning point including the experience they have made (1471): theoretically, by hypothetical thinking, approximation to 2 is understood as leaning to 2 (from potential infinity in the direction of actual infinity). The term *leaning* (*anlehnen*) is non-conventional in this context also in German, the language of the students. Explaining why causes confusion, a deep personal limitation. The interviewer’s repeated demand to explain why, makes M express the GEN as the need for certitude “let’s look at the beginning again here” (1477) while T expresses a need for a new construct “it would be best if we had a function equation...” (1478), valuing highly the construct he looks for, compared with the less valuable representation they have “we only have a functional equation just dependent on the variable before” (1480). Directly after that, the students refer to the need for a new construct expressing their willingness: “right, shall we try to discover something like that ‘cause that would be” (1485), “on that depends on x right?” (1486), “Yes, so f of“ (1487).

The above shows the students’ deep involvement accompanied by meaningfulness, ending up with valuing highly what they do not yet have, expressing a *need for a new construct (NNC)* by situational interest. This scene also shows how the students’ interest is caught: The interviewer values aspects as being most interesting. This empowers the students to act: they immediately deepen their involvement, express their personal experience of limitation, which caused their need for a new construct, which in turn empowered their willingness to construct it. Thus, interest is held, since the NNC and situational interest are mutually reinforcing the students’ progress. **REFLECTING ON THE NETWORKING PROCESS**

AiC postulates that without a need for a new construct (Hershkowitz, Schwarz, & Dreyfus, 2001) the process of constructing such a new construct will not be initiated. When trying to identify such a need for a new construct in data sets, we discovered that it was not always clearly identifiable. However, IDS showed that the learning process was driven in such cases by far more general epistemic needs such as
described in this paper as GEN. The previous research also showed that the GEN was closely linked to seeds for later constructing actions. The main contribution of the present paper is to show that a GEN cannot only drive the epistemic process but may lead to a need for a specific new construct.

Another important result is an integrated view of how the individual and social construction of knowledge promote and call for each other. The flow of ideas as a social flow describes the process as an evolution of ideas in the community. This can happen by gathering and connecting pieces of knowledge that may bring to the fore an aspect that can be recognized as fruitful to be further developed, driven by a GEN or even a specific need for a new construct. This may lead to limitations that again may reinforce the expression of a GEN meeting the situational challenge. Hence, GEN and experiencing limitation fruitfully interact.

We conclude this paper with a reflection on the networking process and on conditions that influenced it positively and negatively. The networking process concerned the different roles of the GEN as seen from each perspective. For AiC, the role of the GEN is its relationship to the seeds of constructing and to the emergence of the need for a specific new construct that marks the beginning of the construction process. In IDS, on the other hand, the role of the GEN is related to situational interest, empowering the students to progress in the construction of knowledge. The GEN is transformed into actions progressing within the flow of mathematical ideas.

The networking process was basically enabled by the following features, in which there was a synergy between the two research teams, already at the outset:

a. The research questions asked by the two teams are rather closely related and refer to how knowledge is constructed by means of epistemic processes and what factors influence processes of constructing knowledge.

b. The theoretical frameworks used by the both teams are based on epistemic actions; while these are not identical, they are complementary.

c. The methodologies used by both teams include fine-grained microanalyses of interview protocols. This afforded us the opportunity to consider a single data set at a similar level of depth from different points of view: We used a three stage cross-analysis methodology that helped overcome some of the difficulties listed below: (i) separate analyses by each team; (ii) re-analysis by each team, in view of the other team’s analysis; (iii) common analysis by both teams in meeting.

d. The notion of GEN that emerged from a first stage of networking (the previous study) has become an integral part of both perspectives and hence a catalyst for further stages of networking (the current study). Hence, the role of the GEN has turned from that of a research result into that of a base for further research using both theoretical frameworks in unison.

On the other hand, some differences between the approaches of the two teams caused difficulties that turned out to be fruitful for challenging our cross-analyses:
a. The AiC approach puts the cognitive aspects in the centre, considering social aspects as important but secondary, whereas the IDS approach considers the social aspects to be of primary importance in constructing knowledge.

b. Therefore, there are differences between the natures of the sets of epistemic actions, which turned out to provide complementary insights.

c. The two approaches espouse somewhat different views of what constitutes construction of knowledge, in particular what kinds of knowledge can be the object of a constructing process and how it could be fostered.

Since the two research teams have already been working together for more than two years, we have reached a state of profound mutual understanding. Hence, each perspective contributes to deepen the insight of the other one into the development of the students’ process of constructing knowledge.

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REFERENCES


