This paper aims at characterizing the teaching actions that are used in a primary school mathematics lesson, and their consequences for the learning progression. To proceed, we explore the analytical outcomes of combining two analytical frameworks, namely the Practical Epistemology Analysis of classroom discourse (Wickman & Östman, 2002; Wickman, 2004; 2006) and the triple [Meso; Topo; Chrono]-Geneses featuring didactical transactions in the Theory of Joint Action in Didactics (Sensevy, Mercier, Schubauer-Leoni, Ligozat & Perrot 2005; Ligozat & Schubauer-Leoni, 2010; Sensevy 2010). The analytic approach is exhibited through an empirical sample of a mathematical lesson about the learning of surface area with 4th grade students. Analyses are guided by two questions: 1) testing the PEA for identifying the content learnt in transactions in the case of mathematics; 2) examining how PEA may augment the MTC-Geneses description to characterize the learning progression over time within the teacher’s and students’ joint actions. This later question is crucial to understand the generalization of the students' experience against the teaching techniques that are unfolded by the teacher.

THEORETICAL BACKGROUND

Practical Epistemology Analysis (PEA)

In the Swedish pragmatist approach to science learning, the PEA framework was developed as an analytical tool for characterizing the meaning-making process in science-classroom discourse. This approach features learning as the unfolding of purposeful action and change of habits for coping with reality (Rorty 1991). Cultural practices entail epistemologies, as implicit rules for acting adequately in social groups. In designing and carrying out classroom work, the teacher makes explicit and implicit decisions about the situations that the students will experience. Wickman (2004) suggests that as the curriculum unfolds in the teacher's and the students' actions and discourse, a practical epistemology is shaped. Hence, from the student point of view, learning content is dependent on the epistemologies developed in the classroom, as a set of epistemic and social norms that guide the selection of relevant actions to achieve a purpose. Of course, such norms are tied to the socio-historical traditions embedded in curricula. We do not aim at describing such rules and power relations per se but we seek for the connection between how classroom participants produce meaning and what meaning is produced in a specific practice.
The model of practical epistemology analysis developed by P.-O. Wickman and collaborators relies upon L. Wittgenstein's notion of language-game (Wittgenstein 1967) and J. Dewey's theory of *continuity in experience* transformation (Dewey 1938/1997). For the former, meaning is a given in the socially shared rules supported by a language proper to a context. Learning is then mastering a language-game, i.e. the grammar of actions featuring a practice. For the latter, experience is continually transformed by the transactions taking place between an individual and his environment. Subjects build continuity between past and present experiences so that experience earned in a given situation becomes an instrument for understanding and dealing with the situations which follow (Dewey 1938/1997, p44). PEA is grounded on *four categories* for analyzing discourse as a transformation of experience within a language game (Wickman & Östman 2002; Wickman 2004; Wickman 2006).

(i) *Encounter*: an encounter delineates a specific situation in terms of what can be seen to meet or interact in actions and discourse. This involves human beings as the participants of the situation and the "things" that become part of the experience in this situation. These may include physical objects, signs, words, utterances, phenomena like natural facts and events as well as recalled experiences.

(ii) *Stand fast*: in an encounter, certain objects are manipulated without any questions arising about their use. Such objects and words stand fast in the encounter. What stand fast in one situation may later be questioned in another situation. Neither does stand fast necessarily imply a correct use from the observer's perspective. It simply implies that the meaning of certain words in discourse is self-evident for the participants with respect to this specific situation.

(iii) *Gaps and relations*: in an encounter participants notice gaps. They then establish difference and similarity relations to what stand fast. If participants succeed in filling a gap with relations they build continuity between past and present experience. If they fail, the gap lingers and the course of action may change direction toward another purpose.

It is important to notice that the four concepts of PEA enable a first analysis of meaning-making from the interlocutors’ perspective. From the observer's perspective (the researcher in this paper), "something" is learnt when the activity moves on, that is when there is evidence that the participants can proceed towards a purpose. Learning proceeds when people notice gaps and fill them with relations to what stand fast in encounters. This inclusive account of learning focuses on what works in the situation in order to overcome it and not solely what is right or wrong with respect to conceptual knowledge. Questioning truth is central in scientific reasoning but it is only one of several ways to proceed in everyday life practices (Habermas 1984/1990). PEA accounts for the meanings being construed in action without prioritizing what is true / not true and what should be said or done in order to acquire the expected knowledge. This must be understood as a methodological caution.
aiming at minimizing the risk of overlooking certain forms of learning just because they were not included in the definition from the outset.

**The triple [Meso; Topo; &Chrono]-Geneses**

The Joint Action Theory in Didactics (JATD) envisions the teaching and learning practices as a *didactical game* in which the teacher achieves his/her goal - making the students learn knowledge content- only if the students get involved and act in *a certain way*. The expected way of acting defines the rules of the learning progression. For the student to learn, the teacher has to design *a set of conditions* made of material and symbolic objects bound to a question, task or inquiry to be attended and featuring the students’ ends in view. This set of conditions is viewed as the "primitive" milieu (or the teaching device) from which the meanings are construed dynamically in action. The teacher and the student(s) cannot achieve their respective purposes without paying attention to the action of the other and moreover to the object of the other's action. An "object" is anything that can be the target of attention of an individual and that can be designated by him. The meaning of an object is given to an individual by the way in which the others are prepared to act toward it (Mead 1934/1992). The teacher's action and the student's action are *joint actions* in which each participant adjusts his action to the other's line of action (Blumer 2004). In the framework of the JATD the triple [Meso; Topo; Chrono]-Geneses models the construction of a *common ground* of meanings in the joint actions that are performed by the teacher and the student(s).

(i) **Mesogenesis** - The fitting of lines of actions of the teachers and the students (or within a student group) to achieve situated purposes generates new meanings through the relations that participants establish to the objects of their environment. In adopting this point of view, the milieu in which actions unfold, is not just the set of conditions defined in the teaching design, regulated by the teacher over time, and against which the student would play a game (the milieu in TSDM; Brousseau 1997, p55-58). It is a constant building up of relationships to objects in discourse and actions, i.e. a mesogenesis in Chevallard's words (1992). Mesogenesis takes up both the students' elaboration of meanings and the teacher's elaboration of meanings on the basis of what the students produce.

(ii) **Topogenesis** – Each category (teacher versus student) lives in distinctive epistemological positions within the classroom collective. These positions are movable but they never merge. The moves in the epistemological positions (either towards a reduction of the gap or towards its deepening) feature the topogenesis. Currently, topogenetic moves result from the division of the activity between the teacher and the students, but also among students themselves according to their potentialities in a specific situation.

(iii) **Chronogenesis** - The teacher knows the overall direction that learning should take on the knowledge timeline. The learning content expected by the teacher in the
first place corresponds to an institutional purpose in terms of contents and values to be conveyed to the students. Such a purpose is described as pieces of knowledge to be learnt, attitudes to be adopted, competencies to develop…etc. in curriculum texts that reconfigure outer-school socio-cultural knowledge historically construed in the human activities. The overall direction that the learning progression takes in the classroom is described by the chronogenesis. Chronogenetic moves result from the legitimating process of certain meanings made by the students in the collective and/or the introduction of new relations directly made by the teacher.

The primitive milieu designed by the teacher is continuously augmented over time by the meanings arising in the participants' fitting lines of action. Meanings are epistemologically distributed across the classroom collective between the students' position(s) and the teacher's position. Certain meanings construed at a time in the mesogenesis may be judged relevant or not by the participants with respect to 1) their potential to support the ends-in-view structured by the task (epistemic relevance); 2) the expected learning content that is the overarching goal of the teacher (didactical relevance). The outcome of the collective analysis of situated actions is the departure point for the participants to further their activities. We contend that the meanings built in the mesogenesis undergo a selective process to become part of a shared common ground in the classroom collective (Ligozat & Leutenegger 2008). The ongoing construction of this reference is an institutionalizing process of meanings construed in the situated actions towards a collective objectivation of knowledge in discourse.

EMPIRICAL FINDINGS
In the following, we attempt to use PEA for describing meanings made in the contingencies of mathematical activities with primary school students. In particular, we try the analytical categories of the PEA approach (encounter, stand fast, gaps and relations) for describing the structure of the mesogenesis. We detail the chronogenetic and topogenetic moves to examine how the teacher directs the students’ attention towards certain relevant objects and correlations in the setting in order to achieve a mathematical task: comparing the surface areas of geometrical shapes.

During the first two minutes, the students cut out the 13 shapes from the cardboard and make conjectures about the use of the objects provided [Gap 1 : what should we do?]. Mark tells that square F and trapezium D are the largest ones. Dina suggests that H is the smaller. Mark sets the shapes F-D-B-C on a line. The word "area" that is embedded in the instruction is not part of the student-student transactions. These first attempts to order the shapes are based on sight estimation. It features the meanings made about the task at this time on the basis of the students' prior experience on ranking objects. When the teacher comes nearby, the students call upon her for helping. The teacher asks to read the instruction and questions the meaning of the
The definition of the word "area" is a purpose prioritized by the teacher. The students suggest that "it is the shapes", i.e. a word that stand fast to them in this situation. But from the teacher's perspective, the students do not manage to construe some relevant relations to the word "area" (something like "it is the surface of the shapes" or "the space lying inside the borders of the shape" may be expected). Hence, Gap 2 lingers and the didactical time stagnates. The following excerpt provides an insight of the teacher's action upon the mesogenesis to build up a common ground about the task to be achieved.

Encounter 1 (line 43-45) ➔ Mark, Dina, Kamer, Teacher take into account shape H and C (among all the set). Gap 1 [what should we do?] still lingers on the basis that certain terms of the instruction do not stand fast to the students. From the words standing fast for the students in the instruction (rank, largest, smallest), the following relation is made to the artifacts: Kamer/ Relation (line 44): we know − smallest − this one − (triangle H). The teacher follows it up and notices a new gap about the term "we know" [Gap 3: How do you know that shape H is the smallest?].

<table>
<thead>
<tr>
<th>Speech turns and significant actions (min 3:29 to 4:20)</th>
<th>Pictures</th>
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| 43. TEACHER: […] it is written/rank from the smallest to the largest///actu-ally from the largest to the smallest | ![PICT-2](image)
| 44. KAM: well///first we know that this one is the smallest (takes triangle H out of the set) | ![PICT-3](image)
| 45. TEACHER: how do you know that is the smallest///how can you prove that it is the smallest of all | ![PICT-4](image)
| 46. MAR: (puts square C close to triangle H [PICT2]) because this one is smaller | ![PICT-5](image)
| 47. DIA: it is half of it | ![PICT-3](image)
| 48. MAR: (murmuring) but we can't prove it | ![PICT-4](image)
| 49. TEACHER: (looking at Mark) OK but…/this is your feelings OK+/+ but how can you prove it+/+/because here (points at the corner of triangle H)/one may say+/+/it sticks out a little bit+/+so how can you prove that it is really the smallest+/+ | ![PICT-3](image)
| 50. DIA: (getting excited) I know I know+ | ![PICT-4](image)
| 51. MAR: oh like this+ according to their area+ we've got to set in line (sets the base of triangle H on the same line as square C [PICT4]) | ![PICT-5](image)
| 52. DIA: no look+ /+I disagree+/+this is smaller because this (points up triangle H) is half of the square (puts H onto C [PICT5]) | ![PICT-6](image)
| 53. TEACHER: ah+ /+do you think this could be a proof+ | ![PICT-7](image)
| 54. [silence 5 sec] | ![PICT-8](image)
| 55. KAM: this+/+that's two+/+mhm+/+the whole square that's twice this one (points at H) | ![PICT-9](image)
| 56. TEACHER: how did Dina do to prove you this+ | ![PICT-10](image)
| 57. KAM: she puts it over (points at H again) | ![PICT-11](image)
| 58. TEACHER: yeah+/+she puts it over the square+/+she superimposed the shapes+/+now+/+you have some transparent paper+/+some square grid paper+/+and by using Dina's technique+/+you've got to find some tricks+/+that to prove that a shape is larger or smaller than another one | ![PICT-12](image)
Encounter 2 (line 45 – 57) ➔ Mark, Dina, Kamer, Teacher take into account shape H and C (among all the set), know, prove, proof. T/ Relation (line 45): you know – you need to prove ➔ a constitutive rule of action. Mark/ Relation (line 46): this one (triangle H) – smaller– (aligning H and square C) ➔ challenged by the teacher (line 49). The notice of Gap 3 by the teacher is a chronogenetical move and it opens a new encounter. The teacher deliberately shifts the attention of the students’ towards a new object: the need to prove the relation [smallest – (triangle H)]. From the students' perspective, the relation [smallest – (triangle H)] stands fast. No one contests that H is the smallest. It is sustained over the succession of encounters but from the teacher's perspective, this relation should not stand fast until it is justified because this is the way mathematical reasoning develops in our culture. In noticing Gap 3, the teacher highlights the relation [you know – need to prove] as a constitutive rule to develop a mathematical reasoning. In order to fill gap 3, Mark uses the relative size of the two smallest shapes (H and C). Dina/ Relation (line 47): it (triangle H) – half – it (square C) ➔ not taken up by the teacher. From the researcher's perspective, the statement H is half of C is a quantification of the magnitude of the surface area of triangle H with respect to square C as the standard. It is an argument to prove that the surface area of H is smaller than the surface area of C. But we cannot ascribe to Dina such a conceptual knowledge about proving practices. The mere thing we can do is to track any correlated relations unfolding in the participants' action and check how these relations are tied in their discourse, whether these relations stand fast in furthering the activity. Mark/ Relation (line 48): can't prove - [H smaller than C] ➔ challenged by the teacher (line 49). T/ Relation (line 49): [it (triangle H) – sticks out – (H and C side by side)] ➔ OPPOSED TO [this one (triangle H) – smaller– (aligning H and square C)]. In fact, the quantification first attempted by Dina does not drive the teacher's attention immediately. The relation made by Mark (line 48) is much more problematic because it counters the teacher's purpose (proving that H is smaller than C). The teacher opposes Mark’s relation (line 46) with another relation that can be made in changing the position of the two shapes: [it (triangle H) – sticks out – (H and C side by side)]. The consequence is that in this position, triangle H may not be smaller than square C. The order relation between H and C changes depending on the geometrical objects considered: side length of square C is smaller than base length of triangle H but side length of square C longer than the height of triangle H. These order relations on length are more or less salient depending on the relative position of the shapes in space (aligning bases PICT 2 or setting them side by side PICT 3). This spatial contingency is used by the teacher to raise uncertainty and make the students noticing [Gap 4: Could C be the smallest?]. Mark/ Relation (line 51): according to area – set in line – (aligning bases of H and C) ➔ challenged by Dina (line 52). Dina/ Relation (line 52): [this (triangle H) – smaller] - because – [this (triangle H) - half of the square - (put H over C)] ➔ OPPOSED TO [according to area – set in line –
Gap 4 is filled in action and discourse by Dina. Gap 4 is a challenge to Mark's belief that we can't prove that $H$ is smaller than $C$ (line 48). Mark attempts to fill Gap 4 in establishing the conditions for triangle $H$ to be smaller than $C$. He tries this in making an inferential relation between the word "area" for the first time at this point and "set in line". The inference is: if the criteria "according to area" (that does not stand fast) was to mean "set in line" (that stand fast in actions), then the conditions for $H$ to be smaller than $C$ would be warranted. Unfortunately for Mark, this inference cannot be retained in the common ground. Dina holds a competitive relation to warranty that $H$ is smaller than $C$ (line 52). If Dina would not have challenged Mark's inference, the teacher would have dismissed it because it does not fit with the expected learning content about surface area. To fill gap 4, Dina brings in a new relation that aggregates previous ones into a causal pattern: [this ($\triangle H$) – smaller] - because – [this ($\triangle H$) - half of the square - (put $H$ over $C$)]. Now the relation ($H$ is half of $C$) is tied in discourse to the order relation ($H$ is smaller than $C$) and it is tied in action to a new pattern of the relative positions of shapes $H$ and $C$ in space ($H$ over $C$). In eliciting [Gap 5: is Dina's utterance a proof?], the teacher empowers the group with the task to self-assess Dina's relation with respect to Gap 3 [How do you know that shape $H$ is the smallest?] and gap 4 [Could $C$ be the smallest?]. Gap 5 is a divert way to make the students aware that Dina's utterance is the kind of proof they seek for, without actually designating it directly to the students. In encounter 2, the teacher keeps a companion stance, without assessing the relations made by the students. It gives the way to the students' reflection about what was done and said in order to objectify the relations into a discursive content that will be shared as a common ground. Kamer/Relation (line 55): whole square – twice ($\triangle H$) $\rightarrow$ a resonance on previous relation made (line 47). The word "proof" does not stand fast to the students and gap 5 cannot be filled. The students remain silent first, then Kamer comes up with a new relation (line 55) as resonance on Dina's relation ($\triangle H$ is half the square $C$) in line 47 and 52. Since the forms of the two relations are different, they are produced by two independent recognition processes used by each student. Kamer does not just take up what Dina said earlier but he builds a genuine relation on different premises to those used by Dina. The order relation between $H$ and $C$ is redescribed in terms of space covered by $\triangle H$ onto square $C$. A mesogenetic resonance is not just a relation that recurs, but it is a complementary relation to another one describing the same reality. From the researcher's perspective, Dina's and Kamer's utterances are significant contents learnt with respect to the task. Dina's utterance fills Gap 3 and 4 because it shows that triangle $H$ is included onto square $C$. Kamer redescribed the order relation between $H$ and $C$ in terms of space covered by $\triangle H$ onto square $C$. But what is the significance of this for the participants? T/ Relation (line 56): Dina proved–this [whole square – twice ($\triangle H$)] $\rightarrow$ gap 5 is filled in discourse by the teacher; a content is instituted in the group. Since the students cannot fill gap
5, the teacher transforms gap 5 into [Gap 6: how Dina proved this?] in line 56. Meanwhile, she highlights a new relation [Dina proved – this [whole square – twice (triangle H)]. This is a subtle chronogenetical move that reduces uncertainty about the significance of Dina's utterance (line 52). Indeed, in line 56, the teacher implicitly establishes that Dina performed a proof. Kamer / Relation (line 57): prove – put over (shape H) → Gap 6 is filled in discourse. T/ Relation (line 58): prove - put (shape H) over the square - superimpose the shapes → a strategic rule of action is instituted in the group. Gap 6 focuses on the "how" it is proved, the student's attention is drifted from the "what" is proved. Kamer's relation (line 57) fills gap 6 in eliciting a strategic rule for handling the shapes: to put one shape over another. This rule is taken up "yeah", rephrased "she superimposed the shapes" and labeled "Dina's technique" by the teacher (line 58). The discursive actions unfolded a strong instituting process of the content of the rule in the referential common ground. From now, it is part of the collective experience of the group and supposed to stand fast for achieving the task.

Encounter 3 (line 58) → Mark, Dina, Kamer, Teacher take into account shape H and C (among all the set), know, prove, proof. T/ Relation (line 58): [transparent paper – square grid paper – Dina's technique – tricks] – prove – a shape is larger or smaller than another one → a new definition of the task to fill Gap 1 [What should we do]. The content of Dina's causal utterance is not instituted as filling gap 4 and 5. In fact, we get no evidence that the gaps 4 and 5 noticed by the teacher are indeed filled by the students. The analysis shows us that the teacher does not attend to the purposes of the students in order to work out the necessary meanings to be construed. To the students, the main purpose is to fill Gap 1 [What should we do].

From the succession of gaps highlighted by the teacher in the chronogenesis, we understand that she prioritized the knowledge of the mathematical concepts (what is surface area? what is a proof?) over the relations effectively made by the students in the encounter (ordering the shapes by sight estimation, finding a numerical ratio between the shapes). Of course, the teacher notices these relations upon which the gaps are raised, but she does not notice that these relations construed in situated action remain contingent for the students. Since these meanings are not tested against other examples and tied together in an instituting discourse, "Dina's technique" does not make sense in the collective experience as a generalized content (or knowledge) that can be in turn, a resource for each student. At the end of the interaction with the teacher, we can say that Gap 1 [What should we do] is partially filled with the strategic rule "superimpose the shape" but since the word "area" is not related to this rule, the concept of surface area does not earn significance in action. Thus, the power of this combined analysis lies in its ability to elucidate the meaning-making process from the participants’ perspective (PEA-analysis), and combine this with an analysis of the consequences of the teacher managing the learning progression in certain, specified ways (MTC-Genesis analysis).
CONCLUDING REMARKS

We now discuss the implications of our analysis for the (re)conceptualization of PEA tools in the JATD. PEA empowered us with tools to analyse learning processes in classroom joint action from the participants’ purposes. From this analysis it can be seen that the basic concepts of the lesson (proof and area) do not make sense to the students as part of the purpose of ordering according to size, without being reformulated as different kinds of doings (putting side by side, juxtapose). This demonstrates that the teacher, in joint action with the students, would need to construe relations between those terms which the students are supposed to learn (proof, area) and those that stand fast (putting side by side, juxtapose) in the transaction between the teacher and the students. Here there is no evidence of this in action, and so there is no evidence to the teacher that students have learnt what proof and area means in terms of habitual ways of talking and acting mathematics. [MTC]-geneses augment the analysis in directing analysis on the overall joint action about how the relations sum up (mesogenesis), the role of the student vis-à-vis the teacher (topogenesis), and how the learning progresses over time (chronogenesis). The MTC-analysis is evident in the aspects of teaching selected for PEA analysis related to in the previous paragraph. Together these two frameworks complement each other in the empirical analysis and help us discerning what transactions may be needed to improve the lesson in terms of its purposes. If teaching is organizing "signs" (words, symbols, constellation of artefacts) to make someone learn a content, learning involves making sense of such signs and forms in order to act adequately with respect to the sign-maker/organizer's purposes. But learning cannot be unilaterally controlled by the organization of signs in a teaching design, however genuine it may be. Learning is contingent on the experience of the learner and on the haphazard sequence of events developing in the joint action of the classroom (Hamza, 2010). Meanings arising in encounters are not "controlled" at their source (in the mind of the students) by the teacher but they are shaped in discourse according to a collective process of selection, aggregation and social validation and so needs close empirical examination.

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