This paper focuses on learning processes of prospective elementary school mathematics teachers who are studying a course on functions and graphs. Specifically, we address the questions – "What were the modifications in students' ways of mathematizing about functions?" and "How did the teacher's and students' actions enable and promote modifications in the students' ways of mathematizing?". For this purpose, we analyze the mathematics classroom discourse that developed in class by combining two theories – systemic functional linguistics and the commognitive framework. In this paper we present the method that we have developed and exemplify its use and advantages by analyzing an episode of a mathematics classroom. By doing so, we provide a lens to think about and capture complexities of instruction.

INTRODUCTION

It is widely accepted today to focus research on learning processes in classrooms, rather than just on the outcomes of learning. This raises substantial theoretical and methodological issues. In this paper we suggest a method for analyzing classroom discourse to learn about instructional processes, and specifically, to address questions such as What were the modifications in students' ways of mathematizing about functions? and How did the teacher's and students' actions enable and promote modifications in the students' ways of mathematizing? Specifically, how did the teacher and students organize the mathematical discourse so that peripheral participants (Lave and Wenger, 1991) could become more active participants of the canonical mathematical discourse? For this purpose we draw on two theories that each seems to have the potential to address different, yet complimentary, aspects of those questions - Systemic Functional Linguistic, (SFL, Halliday, 1978; Halliday & Matthiessen, 2004) and the commognitive framework (Sfard, 2008). This combination enabled us to develop a new perspective with which to investigate classroom learning. The data used to exemplify the suggested tool is taken from a study that aims at identifying instructional processes of prospective teachers, attending a functions and graphs course. In the following sections we outline the basic principles of SFL (Halliday, 1978), followed by basic tenets of commognitive framework (Sfard, 2008). Then we describe and exemplify how we use the suggested method.
THEORETICAL BACKGROUND

Systemic Functional Linguistics

According to SFL (Halliday, 1978; Halliday & Matthiessen, 2004), language is a resource for making meaning through choice. This approach is concerned with the analysis of how language is used to achieve certain discursive goals and the analysis of the choices that have been made in any instance of language use (O'Halloran, 2005:61). The sets of possible choices were clustered by Halliday in terms of the functions that they serve (and therefore are called metafunctions): (1) the ideational – the content function of language, what is talked about. This metafunction expresses those things in language such as the objects, actions, and relations, of the world and of our own consciousness; (2) the interpersonal – the participatory function of language, through which "the speaker introduces himself into the context of situation, both expressing his own attitudes and judgments and seeking to influence the attitudes and behaviour of others." (Halliday, 1978:112); and (3) the textual – the organization of the text. This is the metafunction that "makes language relevant. ... it expresses the relation of the language to its environment, including both the verbal environment – what has been said or written before – and the non verbal, situational environment." (Halliday, 1978: 112-113).

That is, any language use, serves three functions simultaneously, constructing some aspect of experience, negotiating relationship and organizing the language in a way that it realizes a satisfactory message (Christie, 2002).

The commognitive approach to study learning

The commognitive framework (Sfard, 2007; 2008) is a socio-cultural approach. Within this framework thinking is defined as an individualization of interpersonal communication, although not necessarily verbal. Discourse is considered a special type of communication, made distinct by its repertoire of admissible actions and the way these actions are paired with re-actions. To emphasize the unity of cognitive processes and communication, the word commognition, a combination of the two, is used to name the framework.

Mathematics, as any academic discipline, may be considered a form of discourse made distinct by four characteristics: words and their uses, visual mediators, routines and endorsed narratives, as detailed below.

Words and their uses. Any professional discourse has a unique vocabulary. Some of the words may be used in other discourses, either in the same way or according to a different definition. Words and their uses are central to a discourse as often they determine what one can say about the world. With regard to the area of functions and graphs, we find words such as slope and function with unique uses in the mathematical discourse.

Visual mediators. Those are the objects acted upon as a part of the communication. While colloquial discourse is mediated mainly by images of concrete objects that
exist independently of the specific discourse, in mathematics, most symbols and other mediators were created mainly for the purpose of communication. Visual mediators of the mathematics discourse include algebraic symbols that mediate ideas such as written numbers and graphs, or other symbols like those that represent variables, coefficients and equality. The mediators used in the communication often influence what one can say about the idea discussed. To illustrate, while solving equations in algebra, students often participate in a different discourse if they use graphs as their visual mediators, or if they refer to the algebraic symbolic equation as their discursive objects.

Routines. A routine is a set of meta-rules defining a discursive pattern that is repeated in similar types of situations. Those rules are the observer's construct as they describe past actions that were noticed by the observer. Although they describe past actions, routines are helpful in learning a new discourse as our ability to act in new situations often depends on recalling one's or others' past experiences. An example for a routine often practiced in mathematics regards finding the slope of a given linear function. The specific mediator chosen for a function (e.g. graphs or algebraic symbols) often dictates the routine chosen for that purpose.

Endorsed narratives. Endorsed narratives are any text that can be accepted as true by the relevant community. Specifically, in mathematics, the endorsed narratives are those narratives that become "mathematical facts". Narratives such as axioms, definitions and theorems are all endorsed narratives, with each of them being derived differently.

Combining the theories

Our focus is on instructional processes – on the processes by which learning is enabled and enacted. For this we seek a theory that views language as a set of choices. Moreover, the unit of analysis relevant for our suggested studies is the discourse itself (or parts of it). These two requirements are met by both theories. In Gellert's (2009) words, the underlying principles of the two theories are 'near enough'. However, whereas SFL focuses on language, that is, on the verbal aspects of discourse, commognition holds a wider view and considers also non-verbal aspects of the discourse (e.g. routines and visual mediators.) In addition, while SFL explicitly distinguishes between the content function, the participatory function and the organization of the text, studies conducted under the commognitive framework focus on making explicit routines, endorsed narratives, words and visual mediators of the discourse. While commognition is a socio-cultural approach that aims at providing a lens to study learning processes, SFL is a linguistic approach may help researchers focus on specific choices of participants' language use that may be overlooked.

For our purposes, each theory has an added, complimentary, value – we wish to distinguish between the three metafunctions, as is called for by SFL, and we wish to identify the various discourse characteristics, as is called by commognition. We
believe that this dual analysis would allow noticing aspects of classroom discourse that were not identified thus far.

**THE METHOD**

In the following sections we present the data to be analyzed, the unique method that we developed to analyze classroom discourse and an example of using this method to analyze classroom discourse. We begin by specifying our research questions in light of the theoretical frameworks that we adopt.

*Research questions*

**What were the modifications in students' ways of mathematizing about functions?** Specifically, what are the modifications and changes in students' use of words, visual mediators, routines, and endorsed narratives, while they participate in a mathematics discourse? The focus of attention here is Halliday's ideational metafunction. We broaden this metafunction to refer not only to what is being said, but also to the actions performed as a part of instruction (e.g. calculations in writing and drawing.)

**How did the teacher's and students' actions enable and promote modifications in the students' ways of mathematizing?** Specifically, what are the actions that the teacher and students perform to organize the mathematics discourse so that peripheral participants could become more active participants, with regard to the ways by which they use words, visual mediators, routines, and endorsed narratives? How do the teacher and students develop social relationships and how do the participants orient themselves to the learning of mathematics and to others? Here the focus is dual – first, the textual metafunction, that makes language relevant (Halliday, 1978: 112-113). We use this category to refer to what it is that one (usually the teacher) assumes others (usually the students) already know and what it is that is assumed new, and therefore – how the teacher organizes the discourse so that students, as novice participants, could participate. The second focus is the interpersonal metafunction, which regards the ways by which the teacher and her students develop social relationships and by which they orient themselves to each other and to the mathematics.

**Data collection and documentation.** The following transcript is an example of the type of data to be analyzed, and of our method of presenting transcriptions. It was taken during a whole class discussion in a 1st year course for prospective elementary school teachers in a college of education in Israel. All 14 lessons were video and audio taped and are used in a larger study that focuses on learning about functions. In the following task, the students were asked to compare the steepness of five segments (see Figure1). Noam suggested that AB was the steepest, and was asked to explain why:
Noam: I, like, used a silly method.

Teacher: What method?

Noam: I said that, like, in one x, like, the segment AB, then in one x it went up a lot.

Teacher: Wait, wait, wait. I want you all to listen to Noam. Noam is trying to explain why, what was the segment that you said is steepest?

Noam: AB

Teacher: Come, show it on the graph. Come, come, come. Noam wants to explain something, and, a method she worked according to. Come, stand where I stand, and with the pen you can show here on the, … you see, the segment AB is here… come, show them.

Noam: [walks to the overhead. She points with her pen towards segment AB.]

Teacher: That means, what are you saying, that you moved from A, from point A towards the positive direction of the x axis in one unit [moves her hand parallel to the x axis along one unit].

Noam: Yes

Teacher: What did you see, that it went up by how much?

Noam: Three y [the teacher moves her hand parallel to the y axis along three units.]

Teacher: That is, y changed by three [the teacher moves her hand along segment AB]

Noam: Yes

Teacher: When x changed by one, y changed by three.

Teacher: As opposed to the other segments… what is it?

Noam: Let's say, in OA and in BC

Teacher: OA, look, in OA, what happens in OA?
Method of analysis and examples of findings. We use the lens of SFL in general, and specifically, that of the ideational, interpersonal and the textual metafunctions to differentiate between the three discourses that are a part of the mathematical classroom discourse – the mathematical\textsuperscript{5}, the social and the organizational. For that purpose, we broaden each of the metafunctions to refer not only to what is being said, but also to the actions performed as a part of instruction (e.g. calculations in writing and drawing). Specifically, we use the textual meta-function to refer to what it is that one (usually the teacher) assumes others (usually the students) already know and what it is that is assumed new, and therefore – how the teacher organizes the discourse so that students, as novice participants, could participate.

For each metafunction, we refer to the words and visual mediators that participants use, the routines that could be identified and the narratives endorsed.

We summarize and exemplify our method in the table below, with regard to the given transcript. We realize that our sayings are limited due to the shortness of the analyzed excerpt.

<table>
<thead>
<tr>
<th>Discourse characteristics</th>
<th>Words and visual mediators</th>
<th>Routines\textsuperscript{6}</th>
<th>Endorsed narratives\textsuperscript{7}</th>
</tr>
</thead>
<tbody>
<tr>
<td>The ideational</td>
<td>The words and visual mediators used in class that relate to the mathematics. Examples: Student: &quot;Three y&quot; (60) Teacher: &quot;That is, y changed by three&quot; [the teacher moves her hand along segment AB] (61) Student: &quot;in one x, it went up a lot.&quot; (51) Teacher: &quot;when x changed by one, y changed by three&quot; (63)</td>
<td>The routines observed in class that relate to the mathematics. Examples: The student refers to a routine by which she decides which segment is steepest. She calls it &quot;a method&quot; (49). She compares the graphs qualitatively (and not quantitatively.) (turns: 49, 51) The teacher compares the steepness of graphs by referring to their slopes (63).</td>
<td>The narratives that are discussed in this learning community that relate to the mathematics. Examples: at this time there is no evidence that the students and teacher endorse the same narrative. There is evidence regarding the narratives that some of them endorsed: Student: &quot;in one x, it went up a lot.&quot; (51) Teacher: &quot;when x changed by one, y changed by …&quot; (63)</td>
</tr>
</tbody>
</table>
| The interpersonal        | The words and visual mediators by which people orient to the mathematics, to Discursive routines that relate to the ways by which people orient to the mathematics, to Endorsed narratives that relate to the ways by which people orient to the mathematics, to themselves or to others, and by which they
themselves or to others, and by which they develop social relationships and a learning community.

Examples: The student evaluates a method that she chose to use to solve a problem: "I use a silly method" (49)
- The teacher marks Noam's method as important for the entire class: "Wait, wait. I want you all to listen to Noam." (52)

The textual

| The words and visual mediators that relate to the ways by which the participants organize the mathematics discourse so that peripheral participants could become more active participants. Examples: - Choosing the task: comparing steepness of graphs, before actually talking about the slope of a graph. "that means, what you are saying…” (57) |
| Discursive routines that relate to the ways by which the participants organize the mathematics discourse so that peripheral participants could become more active participants. Examples: - Repeating her way of comparing steepness of graphs. - Revoicing. (e.g. "that means, what you are saying…", (57) - Realizing the need for a publicly seen mediator to communicate ideas (graphs on overhead.) |
| The narratives that are endorsed by this specific group that help organize and enable the communication between peripheral and expert participants. Examples: "you should work in small groups" "that means, what you are saying…", (57) |

Table2: A method to analyze classroom discourse

FINDINGS

Mathematics classroom discourse interweaves several discourses – the mathematical, social and pedagogical. We identify the mathematical discourse with the ideational metafunction, the social with the interpersonal metafunction and the pedagogical with the textual. That is, for those discourses we focus on each of the table’s rows separately. For each discourse, we consider each of its characteristics separately, by considering the table's columns.
The following findings are restricted to the given episode, and are therefore limited in their scope.

**Mathematical discourse.** This focus exposes differences in the teacher's and students' ways of mathematizing – their use of words, visual mediators, routines and narratives. The student's word use is more colloquial ("it went up a lot") and the ideas expressed are imprecise. In the context of comparing slopes, while the teacher compares slopes by referring to it quantitatively, the students compare them visually.

**Social Discourse.** The teacher empowers students by allocating time and place for them to present their ideas to the other students and by evaluating their work. The teacher's disposition towards learning seems to be that small group discussions promote learning and that understanding why and being able to express that are crucial for learning.

**Pedagogical Discourse.** The teacher performs several actions to organize the mathematical discourse so that the students could become less peripheral participants. Those actions reveal her dispositions towards learning mathematics: e.g. using revoice as a pedagogical strategy; this way she shows respect her students' ideas, yet able to use them as a springboard to present mathematically accepted ways of doing and saying.

**FINAL COMMENTS**

In this paper we suggested combining two theories, SL and commognition, to learn about instructional processes. The suggested combined method is a practical and coherent way of analyzing classroom discourse to study about learning processes. Thus it provides a conceptual framework by which various aspects of classroom discourse could be observed, identified and thought of. In other words, the combined method helped us to "direct researchers' attention to particular relationships in providing meaning for the phenomena being studied" (Silver and Herbst, 2007: 373). As can be seen, it allows one to focus on each of three discourses that develop in class – the mathematical, the social and the pedagogical. Its importance lies at the underlying assumptions that to improve our understanding of mathematics learning and teaching, one should focus on processes rather than on end-results only, and on those processes that take place in classroom rather than in different "laboratory-like" settings.

**REFERENCES**


---

1. Mathematize: participate in a mathematical discourse; "doing" mathematics (Sfard, 2008).
2. This study is supported by the Israeli Science Foundation, no. 446/10.
3. This is coherent with Vygotsky's theory regarding the higher mental functions that appear first in the social plane and are only then individualized (Vygotsky, 1986).
4. The transcript was translated from Hebrew by the authors.
5. The ideational metafunctions refers to what is talked about. For our purposes, we suggest narrowing the scope addressed by this metafunctions and relate only to the mathematical content discussed.
6. To identify a routine we would need longer stretches of text or declarations regarding how one often acts. As we only exemplify a short episode, the routines are generalized from analysis of a larger corpus of data.
7. While narratives may be presented in a single sentence, their endorsement by the class community may be identified only in longer episodes. Therefore, our examples are taken from a larger corpus of data.
8. We do not claim that the ideation metafunction includes only parts of the discourse that are about mathematics. For our purpose we choose only those parts.
9. In the literature, the pedagogical discourse is often used to consider what we refer to as "classroom discourse" (e.g. Christie, 2002). We use it to refer to teacher's choices regarding the organizations of the discourse so that students, as novice participants, could participate.