A STUDY OF PROBLEM SOLVING ORIENTED LESSON STRUCTURE IN MATHEMATICS IN JAPAN

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This paper presents and analyses “Mondaikaiketsu no jugyou”, the problem solving oriented – approach which is a set of didactic techniques with the aim of motivating the students’ positive attitude toward engaging in mathematical activities and fostering mathematical thinking. As an analytical tool, The Anthropological Theory of Didactics will be applied.

INTRODUCTION
Teaching methods were developed differently in Japan compared to other industrialized countries. Hiebert, Stigler and Manaster (1999) analysed, by studying videos from the material in the TIMSS study, differences in teaching methods and interaction in the classrooms between Japan, Germany and the United States. They argue that Japanese teachers emphasize mathematical thinking, in particular, the development of the pupils' attitudes toward and ability to communicate mathematics, rather than mathematical skills as a goal for the lessons and choose problems starting the lesson that can be solved by varying methods developed during previous lessons. This goal is reached by having the students discuss with the teacher and peers on the settlement options. I will call this type of didactic techniques, where students work on whole-class problem solving, for problem oriented lesson structure (POLS).

A basic problem in mathematics education, and in the training of teachers, is to find ways how to organise the classroom work so as to make the students active learners of mathematics, without losing the focus on the mathematical content. Japanese teaching methods, like the ones described, have attracted attention in Sweden lately (Dagens Nyheter, 2009), and it has been discussed as a possible model to develop in Swedish school system.

Kazuhiko Souma is one of the teacher educators/researchers who has proposed and introduced POLS. He calls his method “The problem solving oriented” approach (shortened to PSO; the author’s translation; “Mondaikaiketsu no jugyō”, in Japanese). The PSO approach is a set of didactic techniques, with motivation based on psychological aspects of learning. Its purpose, like POLS, is to enhance the students’ attitude towards engaging in mathematical activities in the classroom. In Japan, there is a tradition of publishing practical books for mathematics teachers as the target group. This inspirational literature, often written by practising teachers, aim to present ideas and concrete lesson plans, based on well-constructed mathematical problems, that connects to the proposed teaching methods (Souma, 1995; Kunimune & Koseki, 1999, Tsubota, 2007). There are number of practical books for teachers in Japan showing examples of
possible lesson plans. Souma has written and edited a number of such books and his method is actively and widely used by teachers in service in Japan, but has received little attention from the academic community, perhaps because of its practical attribute and the lack of clear theoretical base. The lack of theoretical overhead is perhaps part of the appeal, but is certainly also a problem when one is to describe and assess such approaches. As a tool, I will use the anthropological theory of didactics (ATD) developed by Y. Chevallard (2006). ATD supplies a framework for the analysis of how the didactic process relates to and transforms the mathematics taught in educational institutions and the didactic process is described as an organised collective work aiming to construct a mathematical organisation (MO).

I conjecture that PSO could make a powerful impact during pre-service teacher education due to its distinct didactical structure, but the lack of theoretical base is a hinder. The aim of my paper is therefore to present and analyse PSO in relation to other Japanese POLS approaches, where I incorporate ATD as a tool for the epistemological description, with the assumption that, this is useful for the purpose of didactic planning. To illustrate, I will analyse an episode of practical application of PSO in a Japanese classroom.

BACKGROUND TO THE PSO APPROACH
The Anthropological Theory of Didactics

The anthropological theory of didactics (ATD) approaches learning as institutional issues. Mathematics learning can be modelled as the construction, within a context of social institutions of interlinked praxeologies of mathematical activity, which is also called a mathematical organisation (MO) (Chevallard, 1999 in Barbé, et al., 2005). A praxeology is described by its tasks and techniques (praxis), together with its technology and theory (logos). Technology constitutes the tools for discourse on and justification of the techniques and the theory provides further justification of the technology and connections to other MOs.

The process, under which a mathematical praxeology is constructed within educational institutions, is called the didactic process (ibid.). Chevallard proposes to describe it as being organised in six “moments” that can be thought of as different modes of activity in the study of mathematics. The moments are: (FE) the moment of first encounter (or re-encounter) of tasks associated to the praxeology, (EX) the exploratory moment of finding and elaboration of techniques suitable to the tasks, (T) the technical-work moment of using and improving techniques, (TT) the technological-theoretical moment in which possible techniques are assessed and technological discourse is taking place, (I) the institutionalisation moment where one is trying to identify and discern the elaborated MO, (EV) the evaluation moment which aims to examine the value of the MO.
To organise the work of achieving an appropriate MO, control the didactic process, the educator develop a didactical organisation (DO) with techniques to design a didactic process. It is possible to use ATD to describe a didactic organisations, in terms of praxis and logos block, independently of whether the studied DO’s have ATD incorporated as an epistemological model, but in this paper I will attempt to use ATD to describe and motivate the techniques in the DO proposed by Souma. Thus, in a way, propose an extension of the DO named PSO, with a technological-theoretical block from ATD.

Souma, like most Japanese writers of this genre, often points out the need for general didactic techniques, like giving generosity with positive feedback in order to handle the long-term didactic goals, such as “fostering the students to active learners of mathematics”. The motivation is usually taken from a technological-theoretical block, which could be referred to as “didactical common sense”, where the epistemological model is usually a concrete description of the mathematical situation. In this description I will incorporate the “motivational” technology as “qualitative measures” on the didactic process, like the degree of participation in the didactic process. A more problematic, but central, recurring term is that of “mathematical activity”, which is a concept that measures the degree of participation, interest, independence and motivation with which students are carrying out the mathematical work. A technological term I will use is “invigorate the didactic process” to mean, “increasing the activity” of the didactic process.

The PSO lesson template and technological terms

I will here describe PSO in the form of a “lesson plan template”. Souma states that it is instrumental that the PSO approach is applied with the same basic form regularly. The motivation is that familiarity with the situation makes the students feel more secure in participating in the discourse and engaging in the didactic process.

According to Souma’s example from his book (1995), a typical POLS lesson starts with a teacher giving a problem, for instance, “Show that the difference of the squares of two integers that follow each other is equal to the sum of the two numbers \(5^2 - 4^2 = 9 = 5 + 4,\ 24^2 - 23^2 = 24 + 23\ and so on).” The students try to solve the task and some students who have solved the task write their answers on the blackboard. Then the students explain their solution orally. Souma wonders (pp. 103-104) if the students in this situation will feel a “necessity” to reflect upon the task. Furthermore, some students might not get any ideas on how to solve the problem and will therefore become alienated from the discourse. As an alternative, he proposes the following variation: The teacher writes down expressions on the blackboard without any comments; 

\[5^2 - 4^2 = 9,\ 24^2 - 23^2 = 47,\ (-9)^2 - (-10)^2 = -19\]

and asks the students what they can observe. All students are supposed to be
able find such observations, perhaps working in groups. Students may answer “It becomes odd numbers”, “The differences equal the sum of the integers”, “The differences equals the first integer times two minus one”, “The last integer times two plus one”. After the response of the students, the teacher then controls that all proposals are correct on the blackboard and says; “Now we try to prove each of the statements”. Ideally, the formulated problems have many possible roads to solutions: Several students may use the formula for expanding the square of a sum; and several others, using $x$ to the first integer and $y$ for the second integer, the rule of the conjugate.

Souma proposes to use a didactic technique, which I refer to as guessing. One should, regularly, let all students guess an answer, state hypotheses or formulate questions about the phenomena. It is implied that the “guess” is something that all students can participate in. In the example the students are not, strictly speaking, guessing an answer, but they are invited to, discover patterns by themselves, make hypotheses about the phenomena and by implication set their own tasks. By committing to make a guess or a hypothesis, especially in the social context of the class, the student will have a stronger motivation to study the task and follow it up. Thus using guessing will invigorate the didactical process.

In his book (1997), Souma declares that he is inspired by John Dewey’s theory of reflective thinking. Dewey (1933) presents five cognitive phases of problem solving. 1. Recognize the problem. 2. Define the problem. 3. Generate hypotheses about the phenomena. 4. Use reasoning if the hypotheses are viable to solve the problem. 5. Test the most credible hypotheses. Dewey’s theory has a general scope and is applicable to any problem context. It is also concerned with the cognitive dimensions, rather than the didactic process as such. Souma states that educators in mathematics may have a tendency to hurry up to address the later phase to “use reasoning”. In this way, the development of reflective thinking and motivation may be impeded. Souma thus feels that it is necessary to pay attention especially to the first three phases. He expresses that (1997), from Dewey’s theory, we may infer that it is important that we should “(a) have an aim for why we solve the task, (b) feel a necessity to solve the task and (c) have made hypotheses before starting the reasoning process.” (p. 34) Souma also refers to Polya’s (1957) cognitive theories on problem solving and, in particular, Polya’s insistence on the importance of guessing. Polya states that our hypothesis may of course be wrong, but the process of examining the guess should lead to improved hypotheses and a deeper understanding.

The focus on motivation on the first encounter and the exploration, together with the insistence on a well defined mathematical content, is perhaps the point that, most distinctively, sets PSO apart from other proposed DO’s in the POLS tradition. Souma states that the teacher much take care to plan how the problem is presented and how students are supposed to act in relation to the presented
problem. Souma names (1987) the type of tasks a teacher should aim at, “open-closed” tasks. It means that the tasks, apart from stimulate conjecture and application of guessing, should lead to multiple methods of solution etc., be constructed so as to later stimulate a discourse on theory that should stay somewhat focused on the well-defined subject that the teacher aims to cover. In ATD terms one can say the task should be “closed” so as to give a well defined and controlled vector from the (FE), the moment first encounter, to (EX) and (T), and also a predictable outcome during the following discourse, which usually would concern the establishing of the technological-theoretical environment (TT). The task should also be “open”, by giving the student a chance to make individual choices during the exploring, and later give ample material for discussion, so as to invigorate the didactic process.

Souma means that, starting from standard tasks in the ordinary textbooks of mathematics, the teacher can modify parts of the tasks or change the way of stating them as in the example we saw. If the tasks presented during a sequence of lessons, are carefully constructed, it can lead to conjectures, new problems and methods that productively connects the local MO’s covered to more global ones and inspire to technological and theoretical discourses on higher-level MOs. This type of didactic design can be compared to the ideas proposed in Garcia, et. al. (2006) of designing the didactic/study process so that it constructs, in the end, “integrated and connected” MOs.

This insistence on open-endedness of the task is common with the “open approach method” (Nohda, 1991) is a proposed variant of POLS. The open approach method is used and analysed by Japanese educators (Hino, 2007). Open-ended problems often take the form of formulating a mathematical model and will therefore lead to multitude of, problem formulation solutions and answers. The intent is to let students develop and express different approaches and to let them reflect on their own ideas by seeking to grasp those of their peers (Miyakawa & Winsløw, 2009). Souma (Personal Communication, 2010) judges the open-approach method as something that can not be used in everyday school mathematics. Souma states that POLS lessons applying too ambitious open-ended problems might be isolated from ordinary lessons that, for instance, aim to train students’ basic mathematical skills, but Souma (1987) acknowledge this type of projects at the end of a course. Nohda also notifies that “We do the teaching with the open-approach once a month as a rule” (Nohda, 1991, p. 34).

Bosch et al. (2007) have discussed the danger with open-ended activities, which are often introduced at school without any connection to a specific content or discipline. They state that this type of didactic technology suffers the risk of causing the construction of very punctual mathematical organisations, since this is what students are trained to study.

If we return to the lesson template and the example, the teacher should let students who have different types of solutions present their problem in class.
The teacher then leads the class to discuss the reason behind each method and have the students determine which of the techniques they have used and why. This is the didactic technology of *whole class discussion* of solutions, which PSO has in common with POLS in general. The discussion of alternative solutions gives an opportunity to establish and reinforce technological and theoretical components of the MO studied, like in this case, the expansion of the square, the rule of the conjugate and the different use of variables, i.e., steering the didactic process into (TT), where new methods and techniques are approved. The class discussion also serves the purpose of increase participation and invigorating the didactic process.

After this, Souma recommends that the students have an opportunity to reflect upon the mathematical theory. The teacher can point out what they have learned by having a student read out from the textbooks explanations of the theory relevant for the lessons. During this the theoretical reflection, the teacher can steer the didactic process towards, say, (I) institutionalisation or (EV) evaluation. Souma states firmly (Souma 1997) that studies in mathematics should be organised and based on a well-written textbook that gives a clear explanation of the mathematical definitions and theories. The classroom discourse is only one form of the study process, the study of mathematics will always entail individual studies and individual problem solving inside or out of school. Moreover, the textbook allows the students to recognize and get familiar with the theory, which the textbooks usually explain in more full detail. In other words, the *textbook* technique is proposed, for the purpose of further covering of the moments (TT), (I) and (EV).

**A MATHEMATICAL PROBLEM ORIENTED CLASS IN JAPAN**

The following episode illustrates a mathematical problem oriented lesson where the teacher practices the PSO approach. This study take place during a lesson study in grade eight at a lower secondary school affiliated to the School of Education in Asahikawa, Japan, 2009. The teacher is a former Masters student of Souma. The number of students in this class is 40. The lesson is about how to solve a system of linear equations and is the third lesson on this topic. The students have already studied the addition method by solving linear equations obtained from word problems with an everyday life character. The lesson plan was written and distributed by the teacher to us observers beforehand. Posing the mathematical tasks and problems presented during the lesson is common with the POLS based lesson plans, but distinct to PSO is, that it is always written “students possible conjectures” and “students possible solutions”, so that teachers always prepare different didactic responses depending on which act students take (Souma, personal communication, 2010).

In the guidebook of Japanese national curriculum standards “The curriculum guidelines” (2008) for mathematics for Japanese secondary school, a system of
linear equations with two variables is described (p. 90) as follows: “Solving a linear equation with two unknowns is to make clear that this can be done by using a method that eliminates one of the two variables and then solve equations with one unknown, which is a method students already know”. Thus, the didactic transposition of the praxeology “System of linear equations” to the knowledge to be taught in class (Chevallard, 1985 in Bosch & Gascón, 2006), focuses here on the technique of elimination; reducing the pair of two variables equations to one equation with one unknown. Techniques and technological terms present are substitution, row operations, isolation, coefficients, variables, etc. which are collected from the theoretical base of “Elementary algebra”.

The lesson

As the first step, the teacher shows the problem by verbally reading out a system of linear equations; \(\{7x + 3y = 30, x – 5y = 26\}\) and the students are asked to copy this in writing. He asks: “There are two boys, Taro and Jiro, who both solved this problem. Taro said, “I eliminate \(x\)”. Jiro said, “I eliminate \(x\) as well”. Their answers were the same, but their methods of the solutions are different. Today’s task is to consider how they solved the problem differently”. The teacher does not show the techniques; the students must consider the possible techniques, which obviously is not only one.

The teacher gives them a few minutes (“individual thinking activity” – according to the lesson plan) and encourages them to find as many solutions as possible. He states in lesson plan that this is especially meant for the gifted students who find solutions quickly. The teacher picks up two students who have obtained different techniques and lets those two students write their solutions on the blackboard. The teacher asks the class how many of them used the technique one of the two students has used. The students raise the hands and it is 37 of them. The teacher asks what is the name of this technique and gets the answer “the addition method” which the class already learned at the previous lessons. The teacher asks the class how this technique works. A student answers “Change the coefficient to the same and erase one of the variable”. The student who has written the solution on the blackboard explain her reasoning how she has “changed the coefficient”. She says, “\(x\)’s coefficient must be changed, so I multiplied it by 7”. The teacher responds, – “OK, you multiplied by 7 and got the same coefficient for all the \(x:s\)”. He changes his voice tone a little and then asks “And then, (looks around the class) what can you do with the \(x\)?” Several students respond, “We can eliminate the \(x\)”.

They later discuss the other solution technique called the “substitution method”, He inquires again how many of the students came up with an example of that technique (17; –many of them used both methods), and asks for the name of the technique, and then lets the students explain how the technique works. (Some students might already have learned about the technique at “Juku” – a private school offering special classes held on weekends and after regular school hours.)
The teacher later asks if there are any students who found variants of the addition method, with an intention to let the students be aware to variation of techniques of the addition method. One student presents his solution by multiplying with \( \frac{1}{7} \) to \( 7x + 3y = 30 \), instead of multiplying by \(-7\) to \( x - 5y = 26 \). This presentation awakes a big discussion in the class if it is not a bit too complicated. The teacher concludes the discussion encourage the student with: “But it worked? Didn’t it?”

After the class has had this look at the two different techniques, the teacher lets one student read out loud a passage from the chapter in the textbook, explaining the substitution method. The students work out three to five textbook problems using the substitution method from the book. Afterwards, the teacher asks the class “In which types of problem do you use the addition method and in which types do you use the substitution method?” He lets the students write down their reasoning. The students are then encouraged to create several examples of problems they think fit each technique and different proposals are then later discussed.

**ATD analysis of the lesson**

The purpose of this lesson is to introduce the substitution method and compare it with it to the addition method and to show that both methods reduce the system to the one-variable one-equation case. In his lesson plan, the teacher writes that “The aim of the task” is to “make the students find out there is an another method than addition method through mentioning that two boys use different methods”. He asks how to reconstruct the solution of two boys, instead of asking them “Solve this system of linear equations using substitution method”. This is an instance of the Souma’s guessing technique, since all students are assumed to be able to use the addition method and students are requested to make proposals rather than fixed answers. This is also an example of an “open–closed” tasks; with alternative solutions, but a limited number of possible outcomes. As intended, the task steer the didactic process from (FE) to (EX) and (TT), since it is about finding a new technique, where (TT) is mainly covered during the whole class discussion. The task will also entail (T), technical work, since the students should solve the system with the chosen method. The teacher stimulate participation by having all students report which method they have followed. In the discourse, the teacher takes care to make the students use the correct technological terms, like “addition method”, “substitution method”, and the use of “eliminate” rather than “erase”. Much of the same holds for the final task when they are asked to construct suitable problems for each method. None of these tasks are intended in the MO to be taught, but are the result of a didactic transposition with the intention to reinforce the actually taught MO (Barbé, et al., 2005). As proposed by Souma, reflection on theory is carried out when one student read out loud from the textbook. This steers the didactic process to the moment of (I), so the class verifies now what they have done during the lesson.
More (T) is covered when the students work on problems in the textbook.

**DISCUSSION AND CONCLUSION**

One can summarise Souma’s approach as one firmly grounded in the POLS tradition: He argues for the didactic techniques of presenting problems followed by whole-class discussion, theoretical reflection and the use of textbooks. The main difference with POLS in general is the technique of guessing and that Souma stress the need for *open-closedness* when it comes to task construction. Souma’s theoretical motivation is pedagogical and based on Dewey’s model for the “reflective thinker” and Polya’s cognitive theory. They both focus on the cognitive process of individuals of improved hypotheses and a deeper understanding, rather than the construction of knowledge in a social context.

Like ATD, the PSO takes the organisation of the mathematical content of the study process seriously. I make an attempt to describe PSO using the descriptive potential of ATD to describe the DO proposed by Souma and also to use ATD as an epistemological model to describe the learning object and the learning process: The didactic techniques proposed in the POLS tradition suggest a way to *invigorate* the classroom discourse and PSO, in particular, focuses on how to start up the didactic process using the guessing technique by adding the elements of conjecture, construction and choice from the start, stimulate students’ curiosity to tackle with the mathematical tasks. The guessing technique allows all the students in class to *join* the lesson. *Open-closed* tasks have the dual purpose of both *invigorating* and *control* the didactic process. Some qualitative predicates, like “participation”, “activity” and “invigorate”, regarding the didactic process had to be introduced to cover the motivational issues. Further investigation on how such qualifications properly should be handled is needed. Still, I think that the epistemological components of ATD, fits well as an extension of the *logos* block supplied by the DO proposed by Souma.

**REFERENCES**


