HELPING IN-SERVICE TEACHERS ANALYZE AND CONSTRUCT MATHEMATICAL TASKS ACCORDING TO THEIR COGNITIVE DEMAND

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The focus of this study is on the investigation of seven teachers’ conceptions concerning the cognitive demands of the mathematical task. This study is part of an ongoing research on elementary teachers’ professional development concerning their ability in designing goal oriented activities for mathematics teaching. Through our research, we tried to investigate precise components of teachers’ Mathematical Knowledge for Teaching and Pedagogical Content Knowledge. Teachers collaborated as members of a Community of Practice in order to design, comment and reevaluate mathematical activities, concerning mainly their cognitive demand.

Keywords: mathematical task, cognitive demand, professional development

INTRODUCTION

In our ongoing research we investigate ways to support elementary pre service and practicing teachers in developing a much deeper understanding of mathematics curriculum via problem posing activities.

Research indicates that success of any educational reform depends mainly on teachers. Teachers use textbooks in different ways. Some tend to follow the textbook almost as a script for instruction (Remillard, 1992), whereas others adapt textbook activities and instructional suggestions to the needs of their classroom (Stake & Easley 1978). Researchers, point on these different attitudes drawing the distinction between designed and enacted curricula (Ball & Cohen, 1996). Many teachers function as designers of curricula that are enacted in their classrooms. Nevertheless, traditional use of textbook and teacher-directed approaches dominate (Jaworski & Gellert, 2003), because of a number of socio-cultural issues relating to classroom culture, teachers working individually, textbooks’ structure in discrete lessons, teaching for exams and grading, teachers personal epistemologies, and teachers lack of knowledge. Designing a mathematical unit of study, teachers must first clearly understand the interrelationships of the various ideas within the unit, and second be able to choose or to construct learning activities in order to help students see and appreciate these connections.

In order for teachers to be able to act creatively concerning curriculum, they need support on a conceptual and on an attitude level. To change the way they teach mathematics, teachers must have opportunities to learn mathematical content and pedagogy in new ways, and believe to their capacity to implement the changes.
Concerning knowledge teachers need to teach mathematics, recent research has specified Shulman’s categories (1987) of content knowledge and pedagogical content knowledge subdividing them into common content knowledge and specialized content knowledge, on the one hand, and knowledge of content and students and knowledge of content and teaching, on the other (Hill and Ball, 2004; Ball, Thames & Phelps, 2008). Ma (1999) in her study of the differences between U.S and Chinese teachers pointed to four aspects of knowledge-for-teaching. These are:

- knowledge of basic mathematical ideas
- the ability to make connections between these ideas
- the capacity to create and use multiple representations of these ideas in teaching
- deep knowledge of the curriculum continuum.

Second and third aspect of Ma’s knowledge taxonomy, is similar to specialized content knowledge (SCK). “Perhaps of most interest to us is evidence of the second category — specialized content knowledge. Like pedagogical content knowledge it is closely related to practice, but unlike pedagogical content knowledge it does not require additional knowledge of students or teaching. It is distinctly mathematical knowledge, but is not necessarily mathematical knowledge familiar to mathematicians” (Ball et al. 2008, p.394). This kind of knowledge is of special interest if we want teachers not only to apply but also to be able to choose or construct mathematical learning tasks with high cognitive demand. By cognitive demand we mean the kind and level of thinking required of students in order to successfully engage with and solve the task. For example, tasks as Martha’s Carpeting Task and Fencing Task (Stein et al., 2000) may help students think of fractions, decimals, and percents as different but equivalent representations of rational numbers, but are tasks of different cognitive load. At the heart of SCK lies the skills and knowledge required to unpack, to “decompress” a mathematical concept or skill into its sub concepts. And “decompression” of mathematical knowledge is a prerequisite of specifying and formulating curricular goals and designing corresponding learning activities (Ball et al., 2008; Hill et al., 2008).

OUR RESEARCH

Our research, deals with the professional development of elementary and middle-school teachers. The whole project foresee two phases: Design/Teachers’ Formation, and Implementation. In the present paper we are referring to the first one.

Having in mind that the effectiveness of a lesson depends significantly on the care with which the lesson plan is prepared, during first phase we designed a seminar in order to support elementary teachers on formulating instructional goals and on assessing and constructing mathematical tasks in terms of their cognitive demands.
Planning phase is the most important moment of instruction, because it is during this phase that “teachers make decisions that affect instruction dramatically. They decide what to teach, how they are going to teach, how to organize the classroom, what routines to use, and how to adapt instruction for individuals” (Fennema & Franke, 1992, p. 156).

The main question guiding our seminar’s planning was: How can we help teachers improve their capacity to plan (and enact) lessons that support students’ learning?

Four main competences we anticipated that teachers acquire through seminar. These competences are referred as important for teachers’ professional development by a number of researchers (Stigler & Hiebert, 1997):

- Designing and assessing mathematical tasks of different cognitive demands
- Connecting mathematical tasks with learning goals
- Generating questions that could be asked to promote student thinking during the lesson, and considering the kinds of guidance that could be given to students who showed one or another types of misconception in their thinking
- Anticipating solutions, thoughts, and responses that students might develop as they struggle with the problem

Researchers (Stein et al. 1996; Stein et al., 2000) distinguished four categories of tasks related to cognitive demand: memorization tasks, procedures without connections, procedures with connections, and doing mathematics. They characterized the first two categories as Math Tasks of Low Level Cognitive Demand (LLCD), whereas the last two as Math Tasks of High Level Cognitive Demand (HLCD). The kind of tasks teachers use, largely define what students learn (Hiebert & Wearne, 1993). But, between designing mathematics tasks and implementing them in the classroom intervene many parameters. Further research on cognitive demand of mathematical tasks has sawn that mathematical tasks alone do not guarantee students' learning because teachers often may not implement challenging tasks as they were intended. Stein et al. (1996) found that only half of HLCD tasks are treated as such in the classroom. Investigation of parameters that influence teachers’ decision to maintain or change the cognitive demand of an activity was one of the main goals of the second phase of our research.

In this paper we present instances from the seminar. More precisely, we comment teachers’ difficulties in assessing mathematical tasks, focusing on the investigation of their conceptions concerning the cognitive demands of mathematical tasks

**METHODOLOGY**

Eight in service elementary teachers (2 men and 6 women) participated in the course. Helen, Marianna, and Martha have 20, 22 and 23 years teaching experience in primary schools, Vaso and Despina have 14 and 16 years teaching experience and
Maria, Nikolas and Dionisis have 7 years teaching experience each.

Teachers participated on a volunteer basis, and thus, this is not a random sample and may not be representative of the population of in-service elementary teachers. Nevertheless, as mentioned before we tried to engage teachers from three different groups concerning their teaching experience. Although all were graduated from a University Department of Primary Education they were not familiar with the ‘philosophy’ of new mathematics textbooks. For example discussing during seminar about tasks involving critical thinking, most of them considered that the request for justification is just enough to raise the cognitive load of a task even in the case that the task in an algorithmic routine one. i.e. “Are the fractions 1/2 and 5/10 equivalent? Justify your answer.”

For seminar’s design, we used (lightly modified) the 4-I Model (Teacher-Innovator Model) (Yeap 2006), a teacher-development model for good practices. The model comprises four stages: Ignoring, Imitating, Integrating and Internalizing.

**At Level 0 (Ignoring)** during three approximately two-hours meetings we presented and discussed with the team of teachers three theoretical Frameworks concerning Instructional Design: The “Understanding by Design” (Wiggins & Tighe, 2005), The “Learning by Design” (Kalantzis & Cope, 2005) and “The Implementing Standards-based Mathematics Instruction: A Casebook for Professional Development Ways of Knowing in Science Series” (Stein et al, 2000).

Wiggins & Tighe (2005) describe an approach of designing the teaching unit, focusing on Understanding. They have developed a theory, presenting a multifaceted view of what makes up a mature understanding, the «six facets of understanding»: explanation, interpretation, application, perspective, empathy, and self-knowledge.

Kalantzis & Cope (2005) describe eight «knowledge processes» which represent a range of different ways of making knowledge. Each knowledge process means something different in the structuring of the learning activities. These knowledge processes are: Experiencing the known—or reflecting on our own experiences, interests and perspectives. Experiencing the new—or observation of the unfamiliar, immersion in new situations, reading and recording new facts and data. Conceptualising by naming,—or developing categories and defining terms. Conceptualising with theory—or making generalisations and putting the key terms together into theories. Analysing functionally—or analysing logical connections, cause and effect, structure and function. Analysing critically—or evaluating critically your own and other people’s perspectives, interests and motives. Applying appropriately—or applying insights to real-world situations and testing their validity. Applying creatively—or making an intervention in the world that is truly innovative and creative and that brings to bear your life’s interests, experiences and aspirations.

Neither of these two Frameworks is especially designed for mathematics classroom.
Nevertheless, we considered them as important knowledge source for the seminar, because of their emphasis on teaching and learning by understanding.

**Level 1 (Imitating):** “The Implementing Standards-based Mathematics Instruction”, framework was the main tool of our formatting process. More precisely, after presenting and discussing with teachers the Task Analysis Guide (Stein et al, 2000), we spent one three-hours meeting in commenting and sorting mathematical tasks as of their cognitive demand. At the end, we gave them for reflection the following activity: Choose five activities from your class textbook (or from any other mathematics textbook you wish): 2 of LLCD and 3 of HLCD. For each one of these activities, try to answer the following questions:

1. **Do the activity.** What are all the ways the task can be solved?
2. **What is the cognitive demand of the activity?** (Choose one of five categories of the Task Analysis Guide). Justify your response: In what kind of thinking processes does the activity engage students?
3. **Identify the mathematical goal(s) of the activity.**
4. **What mathematical ideas does the activity develop?**
5. What misconceptions might students have? What errors might students make?
6. **What are the possible difficulties** students may be confronted? Could you anticipate their possible errors?
7. What questions will you ask to focus their thinking? **Identify specific questions through which you could activate students thinking process** concerning the activity (especially in case they are stuck)
8. (In case you think it is necessary) **How you would modify this textbook activity?**
9. **Design, you yourself, an activity of HLCD with the same mathematical goal.**
10. **Write anything else you think as important about this activity**

They had at their disposition one week to reflect and react.

At **Level 2** (Commenting/Integrating) teachers’ reaction to previous questions became the object of a team-discussion during one three-hours meeting. During this phase we tried to apply in our community of practice the six elements of a “Learning by Inquiry Process” as it is described in (Grevholm, 2009), i.e. Teachers are encouraged to ask questions (Questioning), to investigate each others ideas and collect information in order to reformulate their own ideas (Investigation). In this way new knowledge is created (Creation). Teachers, as members of the community discuss the new knowledge (Discussion), and reflect on their old knowledge and practices (Reflection) Discussion and reflections leads to wondering, which raises new questions (Wondering).

At **Level 3** (Internalizing) we asked teachers to design a teaching unit following the preceding theoretical framework.
In what follows, we focus on the teachers’ responses to the given activity (Level 1) as well as their interaction trying to justify their choices within the team (Level 2). The data consists of the three-hour meeting’s transcribed audio, the researchers’ field notes and the teachers’ written responses to the given tasks. During interaction, each teacher in his turn spoke and argued for every question or subject that emerged during the talks, explaining and illustrating his point of view. One could interrupt the flow of the conversation to add something, to express a different or contrasting view or to ask for a clarification.

**ANALYSIS**

Through the analysis, a number of interesting issues concerning the way that the teachers conceive the mathematical task and its cognitive load emerged.

**Defining the cognitive demand of a mathematical task**

The distinction between mathematical tasks of Low Level Cognitive Demand (LLCD) and High Level Cognitive Demand seemed to be a difficult activity for the teachers. We observed that often there were disagreements about the characterization of an active as a LLCD or a HLCD.

Most of them considered that the discrimination of the level of the cognitive demand of a task depends mainly on the complexity of the involved arithmetical operations. The case of Vaso, who tried to justify her decision of a HLCD task, was representative of the teachers’ tension to focus mainly on the arithmetical operations that the task involves. She argues that “...division between large numbers is very difficult for them (students)”. Particularly, she proposed the following task:

**(Task 1)** In a summer camp there are 60 scouts. The scouts are divided into groups of 15 people and form circles to play games.

a) How many circles are going to form? The scouts will form ...... circles.

b) In each circle there are 3 guides. How many guides are all together?

c) How many people are together scouts and guides?

Vaso argues that the fact that there is a number of arithmetical operations that students are asked to carried out transforms the task to HLCD. On the contrary, Maria, another member of the team, considers this particular task as a LLCD arguing that “....the solution path is predefined. There is no doubt about the solution approach.”

Teachers’ inadequacy in providing all the possible solutions of the mathematical task prompted them to underestimate its “cognitive load”. An illustrative example was the case of Despina who considered that a task involving percents is a LLCD one, as it can be solved applying the specific algorithmic routine that students have already taught. She focused only on one possible solution, ignoring a number of interesting approaches, which involve proportions, rates, fractions, and even the use of the
number line representations. In particular, the task she proposed as an LLCD was the following:

(Task 2) The director of a movie theatre, notice that the usual number of the audience on Mondays is about 70 persons. In order to increase the number of the audience he announced that every Monday for each one of the first 45 tickets a movie poster will be provided free of charge. Next Monday he calculated that the 45 persons who won the movie poster were the 60 percent of the audience.

a) Find out how many persons watch the movie that Monday.

b) The cost of each poster is 2€ and the profit of each ticket is 6€. Comparing the total income of that particular Monday with the income of the preceding Mondays, does it worth continuing this particular promotion for the next Mondays?

Through the interaction with the members of the team Despina figured out that, in fact, there are a number of interesting approaches to the task. Moreover, the members of the team realized that the level of the cognitive load of a task could be raised by the investigation during teaching of a range of possible solutions. In other words, they concluded that the “openness” of a task is a factor that possibly defines the level of its cognitive load. Another factor they also considered was that the placement of the task in the teaching sequence. For example, they argued, the summer scouts camp could be a HLCD one, if it had been used as an introductory to the concept of division. Marianna, referring to the specific textbook, asserted: “actually this is the first mathematical problem involving division that students confronted in the textbook of the third grade”.

Modifying the level of the cognitive load

Teachers’ responses to the inquiry of finding ways of “raising” the cognitive load of a task were initially limited to the creation of more difficult/complex arithmetical operations. For example, Maria suggested in the case of the summer scouts camp task to modify the number of the scouts or the number of people in a group so as to provide a division with a remainder. Nikolas transformed that task by involving a reverse arithmetical operation. An interesting case was Martha who argued that giving students through task 2- a general rule of finding percents applicable to a great number of similar tasks, could make task 2 a HLCD one. Actually, she combined cognitive demand and range of applicability. Martha suggested that in order to rise the cognitive demand of task “I should ask students to explore all the possible solutions, present them in class and justify which one is the best”. Nikolas questioned the term “best” for a solution. He argued that it is appropriate to define the criteria under which students will choose a solution. He claimed that the “easiest” solution is eventually the “best” for the students.

The use of multiple representations and connections between different mathematical concepts in order that the task evolves into a “more advanced mode” was also an idea that teachers discussed. For example, Despina suggested asking students
represent the data of the “movie theatre task” in an empty number line and make estimations about the possible solutions. In her opinion, such an intervention may encourage students to make connections between the different representations of the quantities and possibly conceive the interrelations between percents, ratio and proportions and fractions.

**DISCUSSION - Concluding Remarks**

The project created opportunities for in-service teachers to learn and participate with their textbooks in professional development and provided them with opportunities to change their notions of learning and teaching mathematics. The project also offered the opportunity for collaboration between primary school teachers and university researchers, under the main goal to develop knowledge and practice in the teaching and learning of mathematics, so that teachers in schools have better teaching experiences and achieve better conceptual understandings of mathematics with their students.

The analysis of our data provides interesting issues concerning teachers’ ability to characterize mathematical tasks according to their cognitive load as well as their efficiency to reconstruct/redesign tasks, raising their cognitive demands. In almost all cases, the mathematical content of the tasks as well as their cognitive load were not so obvious for the teachers who restricted themselves to school practices reproducing “well known” techniques. For the teachers we worked with during this project, cognitive demand of a task is not independent of the activity’s goal as it is described in textbook’s instruction. More specifically, the position of the task in the unit is a parameter that influences its cognitive demand. A task that is placed in the start of the instruction in order to introduce the students (for example) to the concept of division can be characterized as high-level demand task. The same task could be considered as a low-level task demanding practice if it is placed in the end of a lesson plan.

Concerning the “raising” of the cognitive level of a task, teachers initially confronted difficulties. Their first suggestions were limited to the creation of more difficult/complex arithmetical operations, modifying an arithmetic/numerical element (data) of the problem. But during the progress of the meeting and with the growing discussion and the interactions within the team, teachers orientated towards more effective ways to manage and organize the task. I.e. they referred to the exploration, presentation and discussion of all possible solutions of a task, the use of multiple representations and connections between different mathematical concepts such as the use of a blank number line and estimations of the solutions.

The goal in this research was not to achieve complete agreement between the team but to provide teachers the opportunity to participate in a thoughtful analysis of the tasks, to emerge their shared interest for discussing the characteristics of the tasks and to raise the level of discussion among participants toward a deeper analysis of
the sorting of the tasks. The point was to encourage teachers to dig beneath the surface in determining the level of thinking required to complete a task, based on the point of researchers (Stein, Smith, Henningsen, & Silver, 2000) who claim that when teachers take the opportunity to analyze the tasks, they become more alert to the potential for slippage between intentions and actions in their teaching.

Specific issues generated lively discussion on topics such as the difference between “level of cognitive demand” and “difficulty” of a task, the factors associated with the decline or maintenance of the level of cognitive demands of mathematical tasks during the implementation phase in the classroom, and construction of goal-oriented high-level mathematical tasks.

Hopefully, the results of this study will provide insight into some substantial issues that form the instruction of mathematics. Understanding the cognitive demand level of the mathematical tasks and the ways to “rise” this level will have potential benefits for teachers in acquiring the competencies needed for a better design and implementation of an effective instruction.

REFERENCES


