LABORATORY ACTIVITIES IN TEACHER TRAINING

Francesca Martignone
Math Department of the University of Modena and Reggio Emilia, Italy

The Laboratory of Mathematical Machines of Modena, supported by the “Regione Emilia Romagna” (Italy), invested the most recent mathematics education researches in the teacher training project MMLab-ER. The training course was unique both in methodology and focus: teachers joined laboratory activities with mathematical machines analysing the interactions (between peer, experts and also with tools) and the cognitive processes involved. The paper presents examples of these activities where teachers construct, and then analyse, different resolution strategies carried out during ruler and compass constructions.

Keywords: Mathematical laboratory, teacher training, resolution processes.

INTRODUCTION

The Project "Laboratory of Mathematical Machines for Emilia-Romagna” (MMLab-ER) [1] aims at facilitating the implementation of a laboratory approach in the teaching and learning of mathematics. The first step of the Project was the set up of a network of math laboratories distributed among five cities in Emilia Romagna region (Italy), followed by the training of in-service teachers (primary, secondary and high school) on laboratory activities with special tools, such as the mathematical machines: reconstructions of tools belonging to the historical phenomenology of mathematics from ancient Greece to 20th century (i.e. curve drawers, pantographs and mechanical calculators) [2]. The training course started with laboratory activities on ruler and compass constructions (the compass is one of the oldest and well known mathematical machines) and continued by introducing other curve drawers and pantographs for geometrical transformations used in history both for mathematical purposes and also for practical purposes. In all these activities it is highlighted how, through appropriate tasks, the mathematical machines laboratory activity can be a suitable environment to develop crucial aspects in the teaching and learning of mathematics: for example the exploration processes, the production and comparison of conjectures and argumentations.

This paper presents some examples of teacher training activities in which teacher educators focus the attention on important learning goals, such as the development of adaptive reasoning: “the capacity for logical thought and for reflection on, explanation of, and justification of mathematical arguments” (Kilpatrick, 2001, p. 107).

THEORETICAL FRAMEWORK

MMLab-ER Project is grounded on a laboratory idea that is well expressed by this metaphor: “We can imagine the laboratory environment as a Renaissance workshop,
in which the apprentices learned by doing, seeing, imitating, communicating with each other, in a word: practicing” (Bartolini Bussi et al., 2004, p. 2) [3]. For this reason, the math laboratory should not be conceived only as a physical space in which teaching practices based on the use of specific technologies are developed, but rather as a teaching methodology. This laboratory idea, linked to the tradition of the European cultural history and highlighted by ICMI (International Commission on Mathematical Instruction) since the last century, is suggested by the Italian Commission for the Teaching of Mathematics in the “Mathematics curriculum for the citizen” [4]. During the training, teachers were involved in this type of mathematics laboratory. The hands-on tools used and analysed in these activities are the mathematical machines.

The MMLab-ER Project is based on the experience gained from MMLab in the laboratory activities carried out in school: in particular researches on epistemological and educational aspects involved in activities with mathematical machines (Bartolini Bussi, 2000; Bartolini Bussi & Maschietto, 2006; Maschietto & Martignone, 2008). In recent years the MMLab research analyzed also the cognitive aspects involved during mathematical machines activities. In particular Martignone & Antonini (2009) studied the interaction between a subject and a mathematical machine using Rabardel theory (Rabardel, 1995). According to Rabardel, an instrument is defined as a hybrid entity made up of both artefact-type components and schematic components that are called utilization schemes. Following these ideas, during the interaction with a machine, we have identified artefact exploration processes and different (utilization) schemes carried out to solve a specific task. This study focused our attention on the importance of analysing these aspects and it was also useful to project tasks on the machines explorations dealing with the analysis of artefact components (how is done) and the genesis and development of utilization schemes (how do you use it? What is done?).

The theoretical framework, used to describe and interpret the different phases of laboratory activities with artefact and the role of the teacher, is the construct of semiotic mediation introduced by Mariotti & Bartolini (2008). In this framework

“the teacher's main roles are the following: to construct suitable tasks; to create the condition for polyphony, eliciting the polysemic feature of the artefact; to guide the transformation of situated “texts” (signs) into mathematical “texts”. In this way the teacher mediates mathematical meanings, using the artefact as a tool of semiotic mediation” (Bartolini Bussi, 2009, p.125).

The Project has provided the opportunity to rethink on these existing researches bringing some innovations. For example, the study of the potential that the laboratory activities with mathematical machines can offer in the genesis and development of students’ exploratory and argumentative processes. Our attention to the study of these processes is grounded into different studies: e.g. the researches collected in Theorems in school: From History, Epistemology and Cognition to
Classroom Practice (Boero, 2007) and the study of exploration processes during problem solving activities (Martignone 2007).

The training project has also taken into account international researches in the field of teacher education, many of which were presented in The International Handbook of Mathematics Teacher Education [5]. In particular, we are in agreement with Watson & Sullivan (2008) research because we designed tasks for teachers that focused on important aspects of mathematical activity “to provide insight in to the nature of mathematical activity” (Watson & Sullivan, 2008, p. 110) and we worked with teachers who faced these tasks reflecting on what they are doing and on what way they could make something similar for their students (classroom tasks).

“We use classroom tasks to refer to questions, situations and instructions that teachers might use when teaching students and task for teachers to include the mathematical prompts many of which may be classroom tasks, that are used as part of teacher learning” (Watson & Sullivan, 2008, p. 109)

LABORATORY ACTIVITIES WITH TEACHERS

Teacher training features

The MMLab-ER involved primary, secondary and high school math teachers [6]. For this reason, it was an important opportunity to foster a dialogue and a discussion between teachers from different school levels. Teachers could share ideas and thoughts about the role of the teacher and the different cultural aspects and contents that emerged from the laboratory experiences with mathematical machines. The training course, designed and managed by the author, exploited this opportunity by offering activities that could be an inspiration to teachers from different types of schools, with the common goal of the acquisition of laboratory methodology, the development of attention on exploration and argumentation processes and on relative verbalization.

One of the purposes of MMLab-ER training was to give room for the dynamic discussion and comparison of solution strategies among peers and experts. In particular we asked to explain the procedures of geometrical constructions in order to understand their roots, motivations and development not only related to the mathematical contents involved, but also to the use of tools. Therefore, the focus is on the analysis of own and others' problem solving processes.

It is important to stress that MMLab-ER training did not wish to give pre-packaged worksheets for classroom activities. The working session aimed at providing ideas and guidelines for possible teaching experiments, which were designed during the course according to the needs and the goals of teachers.

Summarizing, the peculiarities of MMLab-ER teacher training are related to:
The course methodology: during training sessions the teachers are placed, with the obvious differences, in learning situations “acting as students”; they are divided in working groups and joined the discussions orchestrated by a teacher educator. In these activities the tasks are open and the different possible solution strategies are described and discussed with all other colleagues who belong to different schools and grade levels.

- The exploration and use of special hands-on tools coming from the history of mathematics and from everyday life.

- The choice to focus on the verbalization and comparison of problem solving strategies (analysing roots, choices, procedures and arguments of the resolutions).

This article presents an experience carried out during the MMLab teacher training, from individual tasks to collective discussions in which teachers described and analyzed different ruler and compass constructions. It will also shed light on the role of the teacher educator who manages the collective discussion with teachers through specific techniques, such as asking to explain, summarizing and highlighting the implicit of resolutions strategies.

**An example: ruler and compass constructions**

The first mathematical machines used in the MMLab-ER training is the most known: the compass. We wanted to reassess the importance of ruler and compass constructions in mathematics teaching-learning: the compass, in fact, although widely used for practical purposes (e.g. in technical education), is not often analyzed in its foundational role in the mathematics culture (we just think to Euclid's Elements).

Following Rabardel theory, teachers analysed the compass. At first, the physical object with its specific characteristics, and then the utilization schemes that develop under specific tasks, for example to draw circles and the measurement transfer. This first activity set the protocol of exploration that will be the basis of each machines exploration. After analyzing the artefact components and its utilization schemes (answering to the questions: *how is it done? What it does?*), teachers studied the role of structure and movements in order to justify the machine functioning (*why it does that?*). Obviously, in this particular case, the exploration was carried out very rapidly because the compass is already well known by the teachers. The analysis of instrument finished with a problem solving activity guided by the open question: “*What if it does change...?*”. Teachers explored the possible changes of the compass (e.g. with equal or different rots and with extensions) and the existing different types of compasses (e.g. plane compass and the blackboard compass).

After this instrument analysis the teachers faced this crucial task: *To construct an isosceles triangle using ruler and compass.*
From the point of view of contents, the choice of the isosceles triangle was made because it is a figure whose definition and properties are known since primary school and therefore suitable to all course participants and reproducible in different classes (with obvious adjustment). Moreover, the request is deliberately left open (we do not ask: to construct an isosceles triangle given the sides) in order to encourage the genesis of different construction strategies. This activity, carried out by all teachers involved in the Project (about a hundred divided into five provinces) was proposed to focus teachers attention on the following aspects: how and why the same final product (in this case, the isosceles triangle) can have different constructions; the importance of analyzing the theoretical and practical reasons grounding the different choices; the role of artefact components and utilization schemes analysis in the planning and development of the resolutions.

Teachers faced the task individually and discussed the possible solutions in small groups and then collectively. This methodology should foster the verbalization of their processes and argumentations. The teacher educator orchestrated the collective discussion using different techniques: calling teacher to play his/her construction, asking to dictate the procedure highlighting the implicit, comparing different constructions and giving suggestions for other possible constructions.

Below we show and analyse some excerpts selected from a collective discussion in which are presented different constructions of the isosceles triangle. The discussion started after the working group session.

Teacher educator: Who can explain to me his construction? Always step by step so we can reproduce it.

Teacher A: I was lazy and I have only drawn a circle with radius at will, then I connected the circle center to two points belonging to the circumference.

The teacher educator performs the procedure on the blackboard (fig.1)

![Figure 1: Solution A](image.png)

Teacher A: I almost chose the sloped side and then I constructed the triangle.
Teacher educator: Okay, first you gave me a procedure and now, do you want to justify it?
The procedure is correct: I found an isosceles triangle. I know that it is just
an isosceles triangle because …

Teacher A: Because the points I have chosen are on the circumference.
Teacher B: Yes, they are the circumference rays and so they are equal.
Teacher A: But now … I’m asking myself this question: I said at will, but I had to be
careful...
Teacher C: They should not be on opposite sides.
Teacher D: They should not belong to the diameter.
Teacher educator: This is an additional step: asking if...
Teacher A: Then, perhaps in order to be more precise I should have said that I excluded
the points diametrically opposite.

Teacher educator: Yes, I should not take B aligned with O and A. The exploration of the
limit cases is important. With a dynamic geometry software this exploration
could be facilitated […]

As we can see in this excerpt, the teacher educator orchestrates the discussion asking
questions, highlighting the limits and peculiarities of the suggested procedures,
raising ideas, and summarizing.

Now we show a second excerpt in which a teacher describes another construction
and, as before, the teacher educator reproduce that on the blackboard.

Teacher E: I draw a segment, I open the compass at will and I draw a circumference
pointing on the two extremes.
Teacher F: But with the same opening.
Teacher E: Yes, maintaining the same opening.

The teacher educator opens the compass less than half of the segment (selected as
triangle base) and she draws the two circumferences while the classroom starts to
rumor.

Teacher E: Not in this way.
Teacher educator: You told me “as I will”…
Teacher E: Larger.
Teacher groups: More than half.

Teacher educator: Ok, why? In the meantime, let us ask our self, what we said before: my
construction starts from a definition or a property of the isosceles triangle.
What property would you use?
Teacher E: I wanted to use the properties of segment axis.
In this excerpt we notice the attention and the effort involved in highlighting the implicits in teachers’ procedures and in the roots of constructions.

The discussion continues analyzing other constructions (see figure 2-3).

In these collective discussions the teachers see and listen to procedures (made by colleagues or by teacher educator), by asking questions and focusing more on understanding the reasons behind the single steps in relation to mathematical concepts involved and the role of compass. This work leads teachers to reflect on the formulation of procedures, the limit cases, the roots of the constructions and the argumentations that support them.

The teacher educator notices that nobody has used the isosceles triangle propriety of “having two equal angles” and therefore she suggests this task: *To construct an isosceles triangle given one of the equal angles.*

This task is interesting because not all the teachers remember the procedure and, for this reason, the activity is seen as a challenge that they have to face by collaborating and discussing in small groups.

Here is a brief excerpt from a discussion that followed the explanation of the procedures carried out by a teacher (teacher 1) who made the construction shown in fig.4.
The teacher 1 constructs an isosceles triangle cutting the angle with a circle and then re-constructs it as we can see in fig.4.

Teacher 2: But the problem is that…we said it is equal to this (he indicates the isosceles triangle sides identified by the circumference that cuts the angles sides). This is equal to this, so these two triangles have equal sides, so they are isosceles.

Teacher educator: The fact is that you have chosen how to cut the angle with the circumference.

Teacher 2: But in this way they are isosceles, did we construct an isosceles triangle using isosceles triangles?

Teacher educator: Yes, they are isosceles triangles, but it could be done also by using the construction of a scalene triangle. The crucial step is when you construct this arc (she points out the line segment that connects the points of intersection between the circumference and the angle sides).

Teacher 1: As a matter of fact, I constructed two congruent triangles, so now I have the correspondent angles congruent; the isosceles triangles are made in few steps.

Teacher 2: It is true, what interested me was to construct an angle congruent to another and in order to do that it is sufficient to construct two congruent triangles. Naturally for the construction of an isosceles triangle I have to put them in this way (he points that out), if they do not mirror, it does not work.

In the excerpt we see how the construction steps are analyzed and discussed with a teacher who wants some clarification about the key elements that ensure the construction validity.

The whole activity on the isosceles triangle construction (individual task followed by collective discussion) highlights how, even in this simple task, we can use the laboratory approach in order to develop the analysis and the comparison of different solutions, underlining the importance of verbalization and explanation of choices (theoretical knowledge, practices, etc.) and procedures. From the point of view of mathematical contents, teachers realize that even starting from the same definition of isosceles triangle (a triangle that has two equal sides) there are different constructions and cognitive processes involved: i.e. the solutions A and C.

The final discussion dealt with the constructions frequency because more than half of the teachers made constructions like in solutions B and C, few like the solution A, and almost none like the solution D. Teachers thought about these choices and concluded that the first two strategies (B and C) are more frequent because they are used both in technique drawing (“when you learn the construction of triangles, you do so”) and when you draw on squared paper (“these are constructions that we, and our students, usually made using the squared paper: the segment axis are simple and
fast to do”); while construction D has not occurred because is not used frequently (“it has not come to my mind because I never do it”). It should be noted that all these reflections were been useful to teachers in the subsequent phase of teaching experiments design, in particular for the a priori analysis, for the tasks choice and for the management of group discussion.

CONCLUDING REMARKS

The methodology and tasks carried out during the MMLab training course aimed at developing the teacher’s attention to other ways of thinking and awareness of their own resolution processes. The development of this type of competence is not only useful for teachers to analyze the possible solutions of their students (the peculiarities in their thoughts and the possible mistakes or misconceptions), but it opens up the horizons on important mathematics features which are not reduced to symbols manipulation or reproduction of proofs studied on books. The reflection on these processes and on the role of theoretical and practical knowledge was one of the most important guide lines in the laboratory activities designed by trained teachers and developed in teaching experiments during the two years of the Project. Even if the analysis of project results is only at the beginning, because the project ends this year, we already have found a correspondence between the guide lines of the training and the teaching experiments carried out by trained teachers. In these teaching experiments, teachers, fostering individual production and critical observations, have always asked their students to verbalize their solutions, by discussing and sharing their knowledge.

NOTES

1. The MMLab-ER is a two-year project (2008-2010) founded by “Regione Emilia Romagna” and coordinated by M. G. Bartolini Bussi and M. Maschietto. The responsible of the teacher training design and development are F. Martignone and R. Garuti.

2. For information about the MMLab mathematical machines: www.mmlab.unimore.it.


6. The training lasted 28 hours (distributed in seven meeting in each of the five provinces) and was managed by expert teachers or researchers who joined the MMLab group.

REFERENCES


