COGNITIVE STYLES AND THEIR RELATION TO NUMBER SENSE AND ALGEBRAIC REASONING

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The purpose of the present study was to examine the relationship between individuals’ cognitive styles, their mathematical achievement in number sense and algebraic reasoning tasks and the strategies they used while solving these tasks. A mathematical test on number sense and algebraic reasoning and a self-report cognitive style questionnaire were administrated to 83 prospective teachers. The results indicated that spatial imagery, in contrast to the object imagery and verbal cognitive styles, is related to the achievement in algebraic reasoning and number sense. The study also revealed that as prospective teachers’ spatial imagery style increases, the use of conceptual strategies in solving the tasks also increases. Implications of these findings concerning the teaching process are discussed.

Keywords: cognitive styles, number sense, algebraic reasoning, procedural and conceptual strategies

INTRODUCTION

The relationship between various cognitive style dimensions and mathematical achievement attracted the attention of several researchers worldwide for many years (Pitta & Christou, 2009b; Presmeg, 1986a). A number of recent studies (Anderson, Casey, Thompson, Burrage, Pezaris & Kosslyn, 2008; Kozhevnikov, Hegarty & Mayer, 2002) examined the effects of students’ cognitive styles on their mathematical learning, utilizing a new approach to the visual-verbaliser cognitive style dimension. These studies, supported the view that there exist three different cognitive style dimensions, a verbal style as well as two types of visual cognitive styles and indicated that the visual-spatial imagery rather than visual-object imagery is related to success in mathematics.

Despite the fact that there have been suggestions for improvements concerning the cognitive style dimensions, research so far (Anderson et al., 2008; Pitta-Pantazi & Christou, 2009b), focused mainly on the investigation of the relationship between cognitive styles and achievement in geometry, spatial and measurement concepts. What is absent in mathematics research, is the investigation of a possible relationship between cognitive styles, number sense and algebraic reasoning. Most broadly, “number sense” refers to the understanding of numbers and the relations among numbers, whereas “algebraic reasoning” is a process in which students generalize mathematical ideas from a set of particular instances, establish those generalizations through the discourse of argumentation, and express them in increasingly formal and age-appropriate ways (Kaput, 1999). Given that the development of both, number sense and algebraic reasoning is crucial for mathematics learning, it is important to investigate whether some cognitive
variables (such as cognitive styles) will enable us better to understand individual differences in solving problems and tasks that involve these concepts. Consequently, the purpose of the present study was to examine the relationship between cognitive styles, achievement and strategies used while solving tasks that involve number sense and algebraic reasoning.

THEORETICAL FRAMEWORK AND RESEARCH GOALS

Cognitive styles

Cognitive style is an individual preferred and habitual approach to organizing and representing information, which subsequently affects the way in which one perceives and responds to events and ideas (Riding and Rayner, 1998). A number of researchers have proposed a wide variety of cognitive style dimensions, such as visualisers-verbalizers (Paivio, 1971), impulsivity-reflectiveness (Kagan, 1965), field dependency-field independency (Witkin & Asch, 1948a). In the field of mathematics education, the verbaliser/imager distinction was the one that attracted most attention (Pitta & Christou, 2009). According to this view, visualisers rely primarily on imagery when attempting to perform cognitive tasks, whereas verbalizers rely primarily on verbal-analytical strategies.

However, in a recent study, Blazhenkova and Kozhevnikov (2009) suggested that there exist two distinct imagery subsystems that help individual process information in different ways (Pitta & Christou, 2009), the object imagery system and the spatial imagery system. Therefore, research provides evidence for two types of visualisers, the object visualisers and the spatial visualisers. Object visualisers have low spatial ability and use imagery to construct vivid high-resolution images of individual objects, while spatial visualisers have high spatial ability and use imagery to represent and transform spatial relations.

Cognitive styles and mathematics

A number of studies have investigated the relationship between cognitive styles and mathematical achievement (van Garderen, 2006; Kozhevnikov et al., 2002; Presmeg, 1986a). However, their results are often conflicting. Some studies have shown that visual-spatial imagery is beneficial for mathematics and that spatial imagery is an important factor of high mathematical achievement (van Garderen, 2006; Kozhevnikov et al. 2002). On the other hand, other studies showed that students classified as visualisers do not tend to be among the most successful performers in mathematics (Presmeg, 1986a), and a probable explanation for this result is the fact that they considered visualisers as one group without distinguishing between the two types of visualisers mentioned above.

Moreover, findings from such studies revealed also certain areas of mathematics for which spatial imagery is important. For example, Kozhevnikov et al. (2002), conducted a study to compare the use of mental images by the two types of visualisers in solving problems. For this reason, the students were presented with
graphs of motion and were asked to visualise and interpret the motion of an object. Students with object imagery style interpreted the graphs as pictures while students with spatial imagery style constructed more schematic images and manipulated them spatially. It is obvious that students with object imagery style will clearly have difficulty solving mathematics problems that involve graphs (Kozhevnikov et al., 2002). Also, the results of a research on mathematical creativity and cognitive styles, which was conducted by Pitta-Pantazi and Christou (2009), indicated that spatial-imagery cognitive style is related to mathematical fluency, flexibility and originality. In another study of Anderson et al. (2008), on geometry problems with geometry clues matched to cognitive styles, both spatial imagery and verbal cognitive styles were important for solving geometry problems, whereas object imagery was not.

**Cognitive styles, number sense and algebraic reasoning**

Children use imagery in their thinking and the properties of images will have consequences on children’s concepts and reasoning (Piaget & Inhelder, 1971). Some studies examined mental representations or imagery in arithmetic and revealed differences between high and low achievers (Pitta & Gray, 1996; Gray, Pitta & Tall, 1997). Low achievers had a tendency to highlight surface details and emphasized the concrete qualities within situations (focused to the descriptive qualities of numbers). On the other hand, mathematically high achievers concentrated more on the relationships and abstract qualities of numbers (Pitta & Gray, 1996). According to Gray et al. (1997), different perceptions of these objects are at the heart of different cognitive styles that lead to success and failure in elementary arithmetic.

The notion of different cognitive styles leading to divergent outcomes stems from the examination of students’ responses to a range of context-free addition and subtraction combinations (Gray et al., 1997). More specifically, they observed that the less able, who tended to focus on surface details of objects, used mainly counting procedures, and they were facing even much more difficulties due to their choice to follow procedural strategies. On the other hand, the above-average children, who focused on more insightful characteristics of objects, seemed to have a better sense of the concept that enabled them to compress the long sequences of procedures (Gray et al, 1997). The aforementioned strategies refer to procedural and conceptual understanding, respectively. According to Hiebert and Carpenter (1992), procedural knowledge is a sequence of actions and conceptual understanding is the knowledge that is understood and that is rich in relationships.

What must be noted, is that the term “cognitive style” was treated in an “informal” way in the abovementioned studies (Gray et al., 1997; Pitta & Gray, 1996), since no use of a tool for measuring the cognitive styles was evident. However, their suggestions about a possible relation between cognitive styles and arithmetic are in accord with some older studies’ results (Navarro, Aguilar, Alcalde & Howell, 1999; Blaha, 1982). Navarro et al. (1999), found that field independence style relates to
achievement in arithmetic and the study conducted by Blaha (1982), showed that
reflective cognitive style, relates to achievement in arithmetic problem solving.

Despite the fact that there has been a limited number of studies examining the
relationship between cognitive styles and arithmetic operations, there is a lack of
studies examining the relationship between cognitive styles, number sense
(divisibility e.t.c.) and algebraic reasoning. Also, it appears that there are no studies
examining the above relationship with the use of an appropriate cognitive style
questionnaire, that is rooted in more general theory of human information
processing (Kozhevnikov et al., 2002). As a result, the purpose of the present study
was to investigate whether mathematical achievement in number sense and
algebraic reasoning tasks and strategies used for solving these tasks, are related to
specific cognitive style. More specifically we sought answers to the following three
questions: (a) Do cognitive styles (verbal, spatial-imagery, object-imagery) predict
prospective teachers’ achievement in number sense and algebraic reasoning tasks?
(b) Does prospective teachers’ achievement in the aforementioned tasks
differentiate in accordance to their cognitive style profile? (c) Is there a relation
between prospective teachers’ cognitive style and the strategies they adopt in
solving algebraic reasoning and number sense tasks?

METHODOLOGY

Participants

The participants of the study were 83 prospective elementary school teachers. All
participants have taken mathematics lessons during their lower and upper secondary
education. A mathematical test and a self-report cognitive style questionnaire were
administered to participants during two sessions.

The mathematical test

The mathematical test on number sense and algebraic reasoning included 10 tasks
Examples of tasks are provided in Figure 1. A verbal and a pictorial task were
employed to examine students’ abilities in each content area (calculation-estimation,
patterns, number divisibility, relations among numbers and problem solving with
unknowns). Using two different representations (verbal and pictorial), we attempted
to achieve a balanced test among the three cognitive styles that were examined in
this study. Two codes were given to each answer. First, the answer was coded as
correct (success=1) or incorrect (success=0). Then, a second code was given for the
strategy used by the participant to complete the task. In the initial stage of the
analysis many strategies were generated which were later grouped into three general
categories. The first category (strategy=1) contained conceptual strategies showing
that the participant has developed a deeper understanding regarding the concept of
number and the relation between numbers. The second category (strategy=2)
included typical and time-consuming strategies and application of formulas, which
were labeled as procedural strategies. The last category (strategy=3) was used to
code the answers for which no strategy seems to have been used. For example, a
prospective teacher gave the following answer when asked to solve task 1 (see Figure 1): “Yes, (there is there a number divisible by 7 between 12 358 and 12 368) because the difference between the two numbers is ten and there must be at least one number that is divisible by 7” (strategy=1). On the other hand, another participant answered as follows: “I believe that the numbers 12 357 and 12 363 can be divided by 7 because the numbers 57 and 63 can be divided by 7” (strategy=2).

<table>
<thead>
<tr>
<th>Task 1</th>
<th>Task 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Is there a number divisible by 7 between 12 358 and 12 368? (Zazkis &amp; Campbell, 1996)</td>
<td>Find the value of one star.</td>
</tr>
</tbody>
</table>

**Figure 1: Sample Tasks of the mathematical test**

**The cognitive style questionnaire**

The cognitive style questionnaire that was a translation of the Object-Spatial Imagery and Verbal Questionnaire (Blazhenkova & Kozhevnikov, 2009) contained 45 statements and examined participants’ differences in spatial imagery (15 items), object imagery (15 items) and verbal cognitive style (15 items). These items measured experience and preference for each cognitive style dimension. Participants were asked to rate the items on a 5-point Likert scale with 1 indicating total disagreement and 5 indicating total agreement. For each participant, the spatial imagery, the object imagery and the verbal score were calculated by finding the average score of the fifteen items corresponding to each cognitive style. The data were analysed using the statistical package SPSS. To answer the research questions of this study, multiple methods of analysis were performed, including regression analysis, multivariate analysis of variance (MANOVA) and pearson correlation. Descriptive statistics were also used.

**RESULTS**

The results of this study are presented in two sections. The first section deals with the relationship between cognitive styles and achievement in number sense and algebraic reasoning, whereas the second is concerned with cognitive styles and their relationship to the strategies that prospective teachers adopt in solving the tasks.

**Cognitive styles and achievement in number sense and algebraic reasoning**

In order to investigate the relationship between cognitive styles and achievement in number sense and algebraic reasoning, we examined the correlations between prospective teachers’ cognitive styles and their achievement, which are presented in Table 1. As it appears from Table 1, spatial imagery cognitive style significantly correlates with prospective teachers’ total achievement score, with their achievement in verbal tasks as well as with their achievement in pictorial tasks. However, the other
cognitive styles (object and verbal) did not correlate with prospective teachers’ achievement.

<table>
<thead>
<tr>
<th>Cognitive styles</th>
<th>Total achievement</th>
<th>Achievement in verbal tasks</th>
<th>Achievement in pictorial tasks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spatial Imagery</td>
<td>.401*</td>
<td>.358*</td>
<td>.345*</td>
</tr>
<tr>
<td>Object Imagery</td>
<td>-.165</td>
<td>-.216</td>
<td>-.075</td>
</tr>
<tr>
<td>Verbal</td>
<td>-.176</td>
<td>-.190</td>
<td>-.118</td>
</tr>
</tbody>
</table>

*Correlation is significant at the 0.05 level (2-tailed).

**Table 1: Correlations among achievement and spatial imagery, object imagery and verbal cognitive styles.**

Then multiple regression analyses were conducted with criterion (dependent) variables the total achievement score, the achievement in verbal tasks and the achievement in pictorial tasks, and predictors (independent) the spatial imagery, object imagery and verbal cognitive styles. The results of the multiple regressions are presented in Table 2 and provide more information about the nature of the relationships between teachers’ achievement and cognitive styles. It is obvious that only the spatial imagery cognitive style is a statistically significant predictor of prospective teachers’ achievement in number sense and algebraic reasoning, regardless of the mode of representation of the tasks, and it explains a respectable proportion of variance (more than 20%) in achievement in number sense and algebraic reasoning. In other words, as prospective teachers’ spatial imagery cognitive style increases, their total achievement in the test, and their achievement in verbal and pictorial tasks also increase.

<table>
<thead>
<tr>
<th>Cognitive styles</th>
<th>Total achievement</th>
<th>Achievement in verbal tasks</th>
<th>Achievement in pictorial tasks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(b)</td>
<td>(p)</td>
<td>(b)</td>
</tr>
<tr>
<td>Spatial imagery</td>
<td>1.339</td>
<td>.001*</td>
<td>.660</td>
</tr>
<tr>
<td>Object Imagery</td>
<td>-.495</td>
<td>.222</td>
<td>-.390</td>
</tr>
<tr>
<td>Verbal</td>
<td>-.199</td>
<td>.593</td>
<td>-.140</td>
</tr>
</tbody>
</table>

*Statistical significance \(p<0.05\)

**Table 2: Multiple regression analyses with dependent variables total achievement, achievement in verbal tasks and achievement in pictorial tasks, and independent variables spatial imagery, object imagery and verbal cognitive styles.**

Moreover, in order to investigate possible differences between the different profiles of cognitive styles, participants were grouped in eight different groups with respect to their spatial imagery, object imagery and verbal cognitive styles as follows: high/low Spatial, high/low Object and high/low Verbal. The mean scores of each group with regard to their total achievement score are presented in Table 3. The highest mean score corresponds to prospective teachers with high preference in spatial visualization processing and low preference in object visualization and verbal processing (group
6). Also, prospective teachers with high preference in spatial processing (groups 1,3,4,6) have higher scores than those with low preference in spatial processing.

To further investigate the impact spatial imagery has on achievement, prospective teachers were assigned to high and low spatial imagery groups. A multivariate analysis of variance (MANOVA) was conducted with the achievement scores in verbal and pictorial tasks as dependent variables and the preference in spatial processing as independent one. The results of the multivariate analysis showed that there were significant differences between prospective teachers achievement according to their preference in spatial processing (Pillai’s $F_{1,81} = 3.882$, $p<0.05$). More specifically, prospective teachers with high spatial imagery have significantly better achievement scores on verbal and pictorial tasks than prospective teachers with low spatial preference.

<table>
<thead>
<tr>
<th>Cognitive style profiles</th>
<th>N</th>
<th>$\bar{x}$</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>S* O* V*</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. H H H **</td>
<td>18</td>
<td>3.89</td>
<td>1.78</td>
</tr>
<tr>
<td>2. L L L **</td>
<td>7</td>
<td>3.14</td>
<td>1.35</td>
</tr>
<tr>
<td>3. H H L</td>
<td>11</td>
<td>4.36</td>
<td>2.58</td>
</tr>
<tr>
<td>4. H L H</td>
<td>11</td>
<td>4.82</td>
<td>2.72</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cognitive style profiles</th>
<th>N</th>
<th>$\bar{x}$</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>S* O* V*</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. L H H</td>
<td>13</td>
<td>3.08</td>
<td>1.80</td>
</tr>
<tr>
<td>6. H L L</td>
<td>9</td>
<td>5.56</td>
<td>2.65</td>
</tr>
<tr>
<td>7. L L H</td>
<td>9</td>
<td>3.22</td>
<td>1.39</td>
</tr>
<tr>
<td>8. L H L</td>
<td>5</td>
<td>4.00</td>
<td>1.23</td>
</tr>
</tbody>
</table>

*S=Spatial, O=Object, V=Verbal, ** H= High, L=Low
Table 3: Means of achievement score for each cognitive style profile.

Cognitive styles and strategies used in solving the tasks

The correlations among prospective teachers’ strategies and cognitive styles are shown in Table 4. It appears that the spatial imagery cognitive style significantly correlated with the use of conceptual strategies ($r=.310$, $p<0.05$) and not procedural strategies, while the rest of the cognitive styles did not correlate with any type of strategies.

<table>
<thead>
<tr>
<th>Cognitive styles</th>
<th>Conceptual strategies</th>
<th>Procedural strategies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spatial imagery</td>
<td>.310*</td>
<td>-.021</td>
</tr>
<tr>
<td>Object Imagery</td>
<td>-.124</td>
<td>-.092</td>
</tr>
<tr>
<td>Verbal</td>
<td>-.152</td>
<td>-.101</td>
</tr>
</tbody>
</table>

*Correlation is significant at the 0.05 level (2-tailed).
Table 4: Correlations among strategies used in number sense and algebraic reasoning tasks and spatial imagery, object imagery and verbal cognitive styles.

To further investigate the nature of these correlations, we analysed our data using multiple regression analysis with criterion (dependent) variables the “conceptual strategies” and the “procedural strategies” and predictors (independent) variables the cognitive styles. The results are presented in Table 5. As it can be seen, the spatial imagery cognitive style is a statistically significant predictor of prospective teachers’
use of “conceptual strategies” for solving tasks with numbers concepts and algebraic reasoning and it explains the 20% of the variance in the adoption of strategies. We can conclude that as prospective teachers’ spatial imagery increases, the use of conceptual strategies in solving various tasks, also increases. On the other hand, none of the cognitive styles can predict the use of “procedural-conventional” strategies.

<table>
<thead>
<tr>
<th>Cognitive styles</th>
<th>Conceptual strategies</th>
<th>Procedural strategies</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>b</td>
<td>p</td>
</tr>
<tr>
<td>Spatial imagery</td>
<td>.864</td>
<td>.009*</td>
</tr>
<tr>
<td>Object Imagery</td>
<td>-.299</td>
<td>.402</td>
</tr>
<tr>
<td>Verbal</td>
<td>-.184</td>
<td>.576</td>
</tr>
</tbody>
</table>

*Statistical significance p<0.05

Table 5: Multiple regression analyses with dependent variables the strategies prospective teachers use and independent variables spatial imagery, object imagery and verbal cognitive styles.

DISCUSSION

Several studies revealed the important role that spatial imagery cognitive style plays in mathematical creativity, geometry and problem solving (see e.g. Pitta & Christou 2009; Anderson et al., 2008; Kozhevnikov et al., 2002). The present study moves one step further and provides evidence that spatial and object imagery may have different effects on the achievement and on the strategies that children adopt for solving tasks that involve number sense and algebraic reasoning. The results indicated that spatial imagery is the only significant predictor of prospective teachers’ achievement in number sense and algebraic reasoning tasks and that teachers with high preference on spatial processing had significantly higher scores on both verbal and pictorial tasks, than the teachers with low preference on spatial processing. This finding is in line with other studies’ results that examined cognitive styles in relation to mathematical problem solving (Kozhevnikov et al., 2002) and mathematical creativity (Pitta-Pantazi & Christou, 2009).

The present study also revealed that spatial imagery is a significant predictor of prospective teachers’ “conceptual strategies” adoption. It appears that as prospective teachers’ preference to spatial processing increases, the adoption of conceptual strategies for solving the number sense and algebraic reasoning tasks also increases. We can say that prospective teachers with high spatial imagery tend to “see” relations between numbers that others do not and consequently they are “in favour” in using quicker and conceptual strategies that involve “understanding” or “insight”. This is in accord with previous research by Gray et al., (1997), who suggested that individuals may approach mathematics tasks in different ways, depending on their cognitive styles. On the other hand, none of the cognitive styles could predict the use of procedural-conventional strategies that involve “rote” or “senseless” actions. The latter might indicate that the adoption of “procedural-conventional” strategies is more of a result of formal instruction in schools and prospective teachers, regardless of
their cognitive style, adopt the aforementioned strategies only when they cannot efficiently use the “conceptual-short strategies”.

Generally, this study helped us gain an overview of how teachers with different cognitive styles “behave” while solving mathematical tasks that involve number sense and algebraic reasoning. Results of this study have further implications for teachers and researchers as well. Teachers should differentiate their teaching and the problem solving tasks appropriately taking into consideration students’ cognitive styles. More concretely, they should help students’ with object imagery and verbal cognitive styles to tackle with tasks that require conceptual understanding and enhance their competence in using more efficient and “non-conventional” strategies for solving problems. Similarly, researchers might extent the results of this study to examine the impact that cognitive styles have on the learning of number sense and algebraic reasoning. Further, they should investigate whether the verbal and object imagery cognitive styles are beneficial for other mathematical areas and tasks.

Concluding, an interesting proposal for future research could be the investigation of in-service teachers’ cognitive styles and their relation to the strategies they teach to students for solving several mathematical problems that involve different concepts. If a certain cognitive style (e.g. spatial imagery) relates to specific strategies, then possible growth of teachers’ spatial processing could improve teachers’ strategies, which in turn could enhance students’ conceptual understanding concerning several concepts.

References


