ANALYSING CHILDREN’S LEARNING IN ARITHMETIC FROM A SOCIO-CULTURAL PERSPECTIVE

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Process-object duality models and the notions of reification, encapsulation or ‘procept’ have provided a way of examining the use of processes and concepts in learning mathematics. Such duality models are based on neo-Piagetian perspectives and the abstract construction of mathematical objects. As part of a project to promote cooperative learning situations in young children’s arithmetic, mathematical learning is also considered from a socio-cultural perspective. Both theoretical perspectives are explored and illustrated through transcripts from a cooperative learning situation with three children age 6 years old. This is used to examine what opportunities are afforded by cooperative learning situations for reification and to consider hypotheses in relation to the social construction of mathematical objects.

INTRODUCTION

Much research has been dedicated to the development of early arithmetic and children’s progression in the use of calculation strategies (for example Gray, 1991). Such studies have indicated that there are ‘milestones’ that show progression from simple counting strategies (‘count-all’ and ‘count-on’ strategies) and the use of commutativity (‘counting-on from the larger number’) to the use of number facts (additive components) and place value. It has been suggested that lower attaining children rely on counting strategies in addition and subtraction (Gray, 1991) and that a reliance on such strategies could hinder children’s progression to more sophisticated, flexible strategies. The use of flexible strategies requires that children have a conceptual understanding of number and their relations.

Several studies have looked at developing children’s generic thinking skills through cooperative group learning. Key to these are studies that have taught children to talk together effectively (Mercer, Wegerif & Dawes, 1999). These have shown that explicit teaching of talk strategies can increase performance in non-verbal IQ tests and in tests for academic subjects including mathematics (Mercer & Sams, 2006). This paper presents work from a project funded by the Esmee Fairbairn Foundation and carried out with colleagues at the University of Exeter. The project introduced strategies for effective talk to young lower attaining children (ages 6-7). Our premise was that such an intervention would support children’s learning in arithmetic and our aim was to examine the mechanisms of talk in relation to children’s conceptual understanding of number.
CONSTRUCTION OF MATHEMATICAL OBJECTS

Effective mathematical learning is one that supports deep conceptual understanding (Schoenfeld, 2002). Such conceptual understanding is commonly seen as the cognitive construction of objects (such as number, relations and functions). Many researchers, including Dubinsky (1991); Sfard (1991); Gray & Tall (1994), have modelled this construction in terms of a process-object duality. Tall, Thomas, Davis, Gray & Simpson (2000) provided a thorough examination of the differences between these duality models but key to these is the notion of encapsulation (Dubinsky, 1991) or reification (Sfard, 1991).

In particular Gray and Tall’s (1994) and Sfard’s (1991) work points towards an initial focus on counting processes in children’s learning in arithmetic. Sfard (1991) described a reliance on ‘count all’ where children count out each set. For example 3 + 4 becomes 1,2,3 add 1,2,3,4. That is each number is seen as a process. This is distinct from a ‘count-on’ strategy, for example 3 + 4 becomes 3 count on 4,5,6,7. In this strategy three is seen as a cardinal number or object that can be ‘counted-on’ from. Three is reified as an object. Gray and Tall referred to the notion of a ‘procept’ where children see the symbol for an operation as both a process and a concept. In order for children to progress in their use of calculations, they need to see this dual nature.

Much of the process-object duality work has been based on neo-Piagetian constructivist theories. These theories are concerned with “the building (of the notion) of a mathematical object as a cognitive process that involves the learner’s construction of adequate cognitive structures” (Dorfler, 2002, p.340). Within a constructivist perspective discourse is seen to describe mental images of objects. It is possible to “ascertaint whether an individual has constructed a mental object” and how the use of language indicates if an individual is conceiving the object (Tall et al., 2000, p.230).

The project that this paper refers to examined an intervention that encouraged collaboration in mathematics tasks. Such an intervention would be based on neo-Vygotskian socio-cultural theories and the notions of a participatory perspective of learning (Sfard, 2001) in mathematics. Here the focus is on cognition as discourse rather than on the use of discourse to understand specific examples of cognition. Speech is not seen as a window to the inner mind to look at the representations stored there; thought and speech are seen as inseparable (Sfard, 2001). Lerman (2001) has argued that Sfard’s complementary view of communication and cognition does not account fully for the individual and suggested an integrated interactionist perspective. Within this metaphor Lerman proposed that social forces and interactions affect learners and ‘pull’ them into increasing participation in mathematics thinking.

Such socio-cultural perspectives move away from a view of language as a way of processing individual thought and accessing the inner mind to a view where language is seen as a cultural tool for shaping and sharing knowledge. In fact from Sfard’s
(2001) perspective communication is not just helpful in constructing and sharing knowledge of pre-existing mathematical objects, communication is the primary cause for the existence of mathematical objects.

What does this mean for children’s learning in arithmetic? Does the process-object duality still play a role and how does this relate to the notion of collaboration and participation when viewed from a socio-cultural perspective? Previous research has illustrated duality from a socio-cultural perspective with more advanced mathematics (Dorfler, 2002) or pre-school children (Sfard & Lavie, 2005). This paper uses both a neo-Piagetian duality lens and a socio-cultural participatory lens to examine young children’s use of counting strategies and examines how discourse has a role in the abstract construction of objects in arithmetic.

THE STUDY

The project was based on a design experiment (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003). It aimed to set up an intervention based on ‘exploratory talk’ (Mercer et al, 1999) in the teaching of arithmetic with young children (ages 6-7) who were seen as low attainers in mathematics. Exploratory talk, a specific aspect of dialogue, was central to the design. Such talk is typified by “a way of using language effectively for joint, explicit, collaborative reasoning” (Mercer et al., 1999, p. 97). Participants negotiate their understanding through constructive challenging discourse.

It was anticipated that the use of exploratory talk would support children’s learning in arithmetic through collaborative reasoning and negotiated understanding.

Twelve teachers worked with the research team over two school terms to develop strategies to introduce exploratory talk and to trial mathematical tasks. Our aim was to analyse the group interactions and the learning of arithmetic that took place through the introduction of exploratory talk. The teachers were asked to select six focus children in each class and to engage the six children in talk and mathematics activities at least twice a week. This was managed in small groups of three children.

This paper presents excerpts from a group activity of one of the triads: Eric, Lydia and Amy. None of the children were recognised officially as bilingual learners but the teacher had identified them as lower attaining, with low confidence in their mathematics and poorer communication skills than other children in the class. Diagnostic tasks, based on those developed by the Shropshire Mathematics Centre (1996), were carried out with the focus children pre- and post-intervention. The pre- and post-tasks for addition carried out with the three children are shown in Table 1.

In the pre-tests Eric relied on ‘count-all’ strategies. Amy used a combination of ‘count-all’ and ‘count-on’ but also used an incorrect strategy. Lydia used ‘count-on’ strategies but did not always use these accurately. The post-tests still indicated reliance on counting strategies but in viewing progression towards more reified strategies then all three children did make some changes.
Table 1: Calculation strategies used by children in pre- and post-intervention diagnostic tasks.

Lydia was more accurate in her use of strategies, she used known facts and made less errors. Eric used more efficient count-on strategies. Amy used a more efficient strategy of counting on from the larger number. There is no attempt to present these results as evidence that the use of talk has supported children in their calculation strategies in a general sense. We also have to acknowledge that the teaching over the term may have benefited the children even if the talk strategies had not been introduced. However it does indicate that these three children made some progression and that it is worth looking at the mechanisms involved in a cooperative group activity to determine if there is any evidence of the role that dialogue may have had in supporting the changes.

The transcript excerpts are taken from a group activity that took place towards the end of one term’s intervention. The task was to construct a rectangle of fifteen dominoes where each join gave a total of six. The transcript is presented in excerpts with a short review.

**Transcript excerpt 1:**

Teacher: The next one you’ve got to put sideways like that. So the four needs a what to go with it? It’s got to go down there but what will it need? Teacher points to show position on the table

Amy: Two.
Excerpt 1 shows how the teacher models the task. As the children are asked to agree
on which domino to use Lydia changes Amy’s (correct) domino with her own. This
does not seem to be a cooperative act and from the video we can see that Amy sits
back from the task whilst Lydia places the dominoes. Amy continues to observe the
activity and does engage again later (see excerpt 4). Lydia expresses disagreement
with a defiant ‘No’ or ‘No, no, no...’ Maybe Lydia is checking the domino is correct
for herself; in the last line she does agree. We also notice that the children do not
justify the selection of dominoes using verbal reasons. Lydia does however count the
dots to check that the join makes six. Although these points raise questions related to
cooperation and children’s reasoning we can see that the children are offering
solutions, for example Eric states ‘Then we need a six’.

In excerpt 2 Lydia selects an incorrect domino. Eric challenges this and offers a
domino to solve the problem. However he does not justify his choice and Lydia does
not appear to question the use of the six or count the dots to verify.

In excerpt 3 the challenge and offer of a solution is repeated. Again the error is when
one addend is zero and we can only speculate that this causes confusion for Lydia.
Lydia does not appear to verify that the six is correct and accepts Eric’s offer of a
solution.

Transcript excerpt 2
Eric: We need a six, we need a six.
Lydia: Another zero.
Eric: That one I think.
Lydia: We need a one.
Eric: You think that’s going to make a six with a zero? (Lydia shakes head). Well get a six then, get a six like that.
Eric refers to the next domino to go with the zero.
Lydia handles a domino but rejects it.
Eric points to two different dominoes that have a six.
Lydia places a domino with a one next to the zero.
Eric holds up a domino (six and five). Lydia places the domino.

Transcript excerpt 3:

Eric: One.
Eric: Six.
Lydia: Now we need a three.
Eric: You mean that’s going to make a six with a zero?
Lydia: No. (Shakes head).
Eric: Two.
Lydia has placed domino (one and zero).
Eric picks up domino with six but Lydia places a domino with three.
Eric holds domino (six and four).
Lydia removes domino with three and Eric places domino (six and four).

In excerpt 4 Amy has rejoined the group activity. She predicts the correct number of dots and checks by counting them. Eric also confirms the ‘two and a four’. When Amy places the next domino this is challenged by Lydia using a similar question to Eric’s. In this case her challenge is not supported by Eric or Amy.

Transcript excerpt 4:

Amy: Ahhh, I know... two, four. 1,2,3,4,5,6
Eric: Two, four. Two and a four.
Amy: Ahhh, three
Amy picks up domino with four and counts the two dots and the four.
Amy places domino (four and three).
Amy places domino (three and zero).

Lydia: You think that’s going to make a six?
Eric: Yeah.

In excerpt 5 the children complete the rectangle. Lydia counts all the dots on both dominoes in trying to decide if the correct domino has been used but Eric’s counting seems to confirm this for her. As the children place the last domino in the rectangle
Amy states that ‘the two is on there’. It may be that she is referring to the final join (a two has been placed next to a one). Eric points to the use of a two elsewhere.

**Transcript excerpt 5:**

<table>
<thead>
<tr>
<th>Eric:</th>
<th>We need a six again</th>
<th>Refers to the zero on the current domino. Eric points to domino (six and three). Lydia places domino.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eric:</td>
<td>Now we need a three</td>
<td>Eric picks up domino with three and two.</td>
</tr>
<tr>
<td>Lydia:</td>
<td>That’s twelve.</td>
<td>Lydia counts all the dots on both dominoes.</td>
</tr>
<tr>
<td>Eric:</td>
<td>1,2,3...4,5,6.</td>
<td>Eric counts the two lots of three dots. Lydia places domino (three and two).</td>
</tr>
<tr>
<td>Amy:</td>
<td>But the two is on there.</td>
<td></td>
</tr>
<tr>
<td>Eric:</td>
<td>Yeah but look.</td>
<td>Eric points to another domino in the rectangle that has a two.</td>
</tr>
</tbody>
</table>

**ANALYSIS AND DISCUSSION**

One of the problems in analysing learning in mathematics from a discursive approach is how to ‘read off’ what learners are thinking (Barwell, 2009). In being objective the researcher as observer may consider they are detached but, as Barwell pointed out, the researcher is also involved in the discourse. It is possible that the interpretation is one where the children are involved in the discourse as the teacher intended but it is not entirely clear how the children have understood the task. Is the focus on making a rectangle or are they looking at the problem of making six? They do miss the final join (although it is possible that Amy noticed this) but an assumption can be made that the children follow the rule of making joins that add to six up to the final join. Within this constraint an attempt is made to analyse the group interactions in relation to the characteristics of exploratory talk and the children’s learning in arithmetic in relation to the duality model.

From the excerpts above children’s engagement with the characteristics of exploratory talk (cooperating, challenging and justifying) are questioned. Carrying out a concordance analysis of the transcript shows that only the teacher uses the word ‘agree’. There is little evidence of words indicating justification, for example the word ‘because’ is not used. At one point the phrase ‘Yeah but look’ is used alongside pointing. Does this suggest that pointing to an example is offered as a justification? The children also justify the selection of a domino through pointing. It could be that the domino is seen as correct by the other two children so is not questioned. In some cases the children verify that the domino is correct by counting the dots. Once it has
been verified by counting then there is no need to question the use of the domino further.

Although there are occasions of non-cooperation, overall the children do work together to complete the rectangle following the rule. The pronoun ‘we’ is used frequently and within the context of ‘we need’. Eric is also proactive in offering solutions ‘We need a six’ or ‘That one I think’. The children do also challenge ideas. Eric’s use of the terms ‘You think’ and ‘You mean’ when challenging Lydia in her placement of dominoes next to zero, question her position in making the decisions.

The next step is to analyse the mathematical learning. In viewing the activity through a socio-cultural perspective the concern is not with a rationalist, individual acquisition of knowledge but in participation. Sfard (2008) described this as participation in social routines and it is possible to determine the development of routines in the transcripts. For example a problem is initiated ‘So the four needs a what to go with it?’ There is the offer of a possible solution ‘A two’, ‘We need a six’. The children then select the appropriate domino. In some cases the selection is verified through counting, on other occasions the selection is challenged. In fact the challenging becomes a routine. Lydia attempts to imitate this (albeit at an inappropriate point), so is learning to participate even if she does not have the knowledge (Sfard, 2008).

Hence the dialogue suggests elements of developing mathematical routines but is there evidence that the children are constructing mathematical objects from a neo-Piagetian duality perspective. In finding the appropriate domino the children are finding an unknown number, the number that complements to make six. In this way the number is held as an object. This is also modelled by the teacher as she asks ‘So the four needs a what to go with it?’ and treats the unknown number is an object. In this way the teacher uses the indefinite article ‘a’ to indicate the number as a noun and hence an object. Concordance analysis shows that the term ‘We need a...’ (for example ‘we need a six’) is used frequently by the children suggesting that the children refer to numbers as nouns, that is as objects.

Dorfler (2002) indicated that the use of language provides means to express something as an object, such as the use of nouns, and the use of actions on objects, such as the use of verbs. In the routine that challenges the solutions the phrase ‘You think that is going to make...’ is used. Here verbs are used to indicate action on an object. Is the suggestion that a process should be carried out to check the solution? Processes such as counting the dots are used by the children either to select a domino in the first place or to verify the use of a domino. In this way processes are repeated in terms of determining an object and processes are turned into objects as another means to support objectification (Dorfler, 2002).
SUMMARY

Although an initial inspection of the transcripts questions how well the children are engaged in exploratory talk further analysis would suggest that there is a level of cooperation and that this cooperation leads to the use of mathematical routines. There is also evidence that the use of the routines involved the children in use of mathematical objects (‘a what’, ‘a two’ or ‘a six’) and that the children repeatedly refer back to processes. This would suggest that the cooperative learning opportunity does allow for children to use both objects and processes in a repetitive way.

A further question is whether this repetitive use of objects and processes indicates a transition between objects and processes in relation to reification. If so how what is the role of the discourse? It could be said that assertions made by the children are part of the negotiation of meaning and that through such interactions ideas are rephrased and reflected on and processes are offered as objects (Zack & Graves, 2001). So is an individual abstract construction derived socially from a discursive construction?

Within Vygotskian and Wittgensteinian viewpoints a concept is understood through its discursive use (Sfard & Lavie, 2005). Meanings are ‘taken as shared’ and these meanings emerge through interaction. According to Sfard (2008), a mathematical concept is an objectification or reification of a discursive process and the abstraction of mathematical relations can be seen as ‘culturally-shaped’. So when we are looking at ‘what learners are thinking’ are we looking for an individual mental image of number that is ‘described’ in talk or are we looking at a ‘by-product of the discursive growth’ (Sfard & Lavie, 2005, p. 243)? This is not to say that the ‘so-called’ construction of objects is not cognitive. Within abstraction there is an intention to view items and actions as unified and that this intention is a deliberate decision (Dorfler, 2002). This decision has to be made by an individual but within a cooperative learning situation there is an opportunity for the decision to be made within the context of an activity and hence can be seen as socially derived.

REFERENCES


Sfard, A. (2001). There is more to discourse than meets the ears: Looking at thinking as communicating to learn more about mathematical learning. *Educational Studies in Mathematics, 46*(1-3), 13-57.


