SECONDARY SCHOOL STUDENTS’ PERCEPTION OF BEST HELP GENERALISING STRATEGIES

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This paper reports a study whose aim was to examine secondary school student choices of generalising strategy to determine what they would judge as the most helpful strategy for expressing generality. Data were collected from 45 Secondary One students through administering a questionnaire which contained one linear and one quadratic generalising tasks. The students had to select and justify the strategy that they believed would best help them to establish the rules. The data were analysed and revealed differences in the judgements of the more able and the less able students regarding the best-help strategy. The more able students’ choices of best-help generalising strategy seemed to vary across the two different types of tasks whereas those of the less able students appeared to remain unchanged.

**Keywords:** Pattern generalisation, student beliefs, generalising strategy

**BACKGROUND**

Modelling mathematical problems as examples to demonstrate how the problems in question can be worked out appears to be an important teaching activity that features strongly in mathematics lessons. When illustrating examples, teachers tend to introduce their methods or strategies to show students the way to deal with these problems. They could have picked these methods based on their beliefs about what students are capable of understanding and how they will learn best (Chua & Hoyles, 2010b). All this is done with the good intention of wanting to help students experience some success. And students generally accept and follow the teachers’ methods. But some teachers may go as far as to enforce their methods and get students to comply. So the big issue here is what students think of their teachers’ methods. Do the teachers’ methods help them understand and learn the examples better?

For generalisation of number pattern, the literature identified several kinds of generalising strategies used by students to establish a rule between the term and its position in the pattern (Drury, 2007; Lannin, 2005; Lee & Freiman, 2006; Rivera & Becker, 2008; Steele, 2008). In some of these recent studies, the students were even presented with different-looking rules that could be used to describe the same underpinning pattern and asked to justify how these rules could all be equivalent to one another (Drury, 2007; Lee & Freiman, 2006; Rivera & Becker, 2008). Such an activity challenges them to use different strategies to come up with multiple ways of seeing the same pattern. However, none of these studies went further to ask students for the kind of strategy that they believe will best help them to construct those rules.
Thus the present study sought to fill in this gap by examining secondary school student choices of generalising strategy to ascertain what they would judge as the most helpful strategy to establish the functional rule for deriving any term in the pattern. It is hoped that the findings of the present study could provide valuable insights for teachers, teacher educators and curriculum developers.

THEORETICAL FRAMEWORK

Pattern generalising tasks are a common feature of school mathematics in many countries. Generally, researchers concur that such tasks are a powerful vehicle not only for introducing the notion of variables (Mason, 1996) but also for developing two core aspects of algebraic thinking: the emphasis on relationships among quantities like the inputs and outputs (Radford, 2008) and the idea of expressing an explicit rule using letters to represent numerical values of the outputs (Kaput, 2008).

A typical generalising task involves facilities like identifying a numerical pattern, extending the pattern to make a near and far generalisation, and articulating the functional relationship underpinning the pattern using symbols.

There is a wealth of research that examines students’ generalising strategies and reasoning when they deal with pattern generalising tasks. Students had been found to use a variety of strategies for constructing the functional rule underpinning the pattern depicted in the tasks. For instance, Rivera and Becker (2008) established three types of strategy that students employed: (1) numerical, which uses only cues established from any pattern that is listed as a sequence of numbers or tabulated in a table to derive the rule, (2) figural, which only applies in generalising tasks that depict the pattern using diagrams, and relies totally on visual cues established directly from the structure of the figures to derive the rule, and (3) a combination of both the numerical and figural approaches.

Different types of strategies do exist even within the numerical solutions. Bezuszka and Kenney (2008) identified three such strategies that involve recursion: (1) comparison, where the terms in a given number sequence are compared with corresponding terms of another sequence whose rule is already known, (2) repeated substitution, where each subsequent term in a number sequence is expressed in terms of the immediate term preceding it, and (3) the method of differences, also known as finite differences in Mathematics, which is an algorithm for finding explicit formulae that are polynomial equations.

The figural solutions were further distinguished into two different categories by Rivera and Becker (2008): (1) constructive generalisation, which occurs when the diagram given in a generalising task is viewed as a composite diagram made up of non-overlapping components and the rule is directly expressed as a sum of the various sub-components, and (2) deconstructive generalisation, which happens when the diagram is visualised as being made up of components that overlap, and the rule
is expressed by separately counting each component of the diagram and then subtracting any parts that overlap.

Apart from these two kinds of *figural* strategy, Chua and Hoyles (2010a) introduced two other strategies into the existing classification scheme developed by Rivera and Becker (2008). One of them occurs when one or more components of the original diagram are rearranged into something more familiar. This newly reconfigured figure then unveiled the pattern structure and facilitated the construction of the functional rule. The other happens when the original diagram is viewed as part of a larger composite figure, from which the functional rule is generated by subtracting the sub-components from this composite figure.

To sum up, the literature review leads us to recognise the diverse ways of constructing the functional rule that represents the pattern in a generalising task. Hence this present study aims to add to the body of work on pattern generalisation by seeking to answer some of these questions: Which strategies would students believe would best help them to work out the rule? How would more able students’ choices of best-help strategies compare with those of the less able students? If the rule underpinning the pattern were to change from a linear to a quadratic relationship, would the best-help strategies that students considered for the former case change to suit the latter?

**METHODS**

Student data were collected through a questionnaire administered to 45 Secondary One students (aged 13 years) from a secondary school. 29 of the students came from the Express course and 16 from the Normal (Academic) course. The students were placed in these courses based on their performance at a national examination taken at the end of their primary education when they were 12 years old. These students, 22 boys and 23 girls, were selected by the school according to their Mathematics grade in the national examination. Amongst the Express students who were considered academically more able than the Normal students, 15 scored an A or A* (high distinction) for Mathematics while the remaining 14 scored a B or C. All the 16 Normal (Academic) students scored a B or C because no one obtained A or A*.

These students had already learnt the topic of number patterns, which is part of the Singapore mathematics curriculum, before participating in this study. So they should be able to continue any pattern, whether presented as a sequence of either numbers or figures, for a few more terms, make a near and far generalisation and derive the functional rule in the form of an algebraic expression for predicting any term. Further, they should also be far more familiar in dealing with linear patterns than with non-linear ones, which are less common in their mathematics textbook.

Before administering the questionnaire, a worksheet comprising the two generalising tasks that were used in the questionnaire was distributed to every student. The two tasks, *High Chair* and *Christmas Party Decoration*, are presented in Figures 1 and 2.
respectively below. The first task involves a linear rule whereas the latter involves a quadratic rule. These two tasks differ from the typical textbook tasks in that they are less structured, thus allowing a greater scope for exploring the pattern structure. The students were asked to work out the functional rules in terms of the size number individually using any strategy that they were familiar with. The purpose was to prepare and familiarise them with these tasks so that they could better understand the questionnaire tasks that they had to do later.

![Figure 1. High Chair](image1)

![Figure 2. Christmas Party Decoration](image2)

Subsequently, the questionnaire containing those two generalising tasks, each accompanied by four possible student solutions, was distributed to each student. Figures 3 and 4 below show the four distinct student solutions for the two respective tasks. Set in a context of a discussion amongst four students, each student solution represented a different way of constructing the rule based on the classification scheme described above. Take, for instance, the solutions in *High Chair*. Method 1 involves rearranging the original figures into something more familiar (S3). In
Method 2, the original figures are viewed as part of a larger rectangle with four missing cards (S4). Method 3 uses a *numerical* strategy (S1) known as the repeated substitution strategy (Bezuszka & Kenney, 2008) while Method 4 employs a *constructive* strategy (S2). For *Christmas Party Decoration*, Methods 1, 2 3 and 4 correspond to S4, S2, S3 and S1 respectively. The students were asked to choose the method that they believed would best help them to construct the functional rule. In addition, they had to provide justifications for their choices of the best-help method.

**Figure 3. Student solutions to High Chair**

All 45 questionnaires were collected and analysed to determine the student choices of method that they thought would best help them to work out the rule. The frequencies of the four student methods for each generalising task were then counted. The student justifications were looked into to gain a better understanding of the reasons behind their choices of best-help strategies.
RESULTS

This section presents the findings to the following two questions that guided this study.

1. Which strategies would students believe would best help them to work out the rule for High Chair?

Table 1 shows that the numerical solution S1 was the top choice of best-help strategies amongst the Express students in this study, with 13 of them selecting it. Following it, in descending order, are S2, S4 and S3. There were nearly an equal number of students choosing S2 and S4, with another three preferring S3. Taking these numbers of students collectively, 16 of the 29 Express students believed that a figural method would best help them to derive the rule. Similarly, a significant number of the Normal (Academic) students (69%) also found the numerical method S1 most helpful. As for the rest, three chose S2, two selected S3 and none opted for S4.

<table>
<thead>
<tr>
<th>Method to students</th>
<th>Best-help Method</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>High Chair</strong></td>
<td>S1   S2   S3   S4   Total</td>
</tr>
<tr>
<td></td>
<td>S1   S2   S3   S4   Total</td>
</tr>
<tr>
<td><strong>S1</strong></td>
<td>13   4    2    0    19</td>
</tr>
<tr>
<td><strong>S2</strong></td>
<td>12   10   1    0    23</td>
</tr>
<tr>
<td><strong>S3</strong></td>
<td>11   8    0    0    19</td>
</tr>
<tr>
<td><strong>S4</strong></td>
<td>0    0    0    0    0</td>
</tr>
</tbody>
</table>
Table 1: Student Choices of Best-help Method for High Chair

<table>
<thead>
<tr>
<th></th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Express Students</td>
<td>7</td>
<td>13</td>
<td>4</td>
<td>5</td>
<td>29</td>
</tr>
<tr>
<td>(n = 29)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Normal (Academic)</td>
<td>12</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>16</td>
</tr>
<tr>
<td>students</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(n=16)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

S1: Numerical; S2: Constructive; S3: rearranging the original figures; S4: viewing the original figures are viewed as part of a larger rectangle

Table 2: Student Choices of Best-help Method for Christmas Party Decoration

<table>
<thead>
<tr>
<th>Christmas Party Decoration</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Express Students (n = 29)</td>
<td>7</td>
<td>13</td>
<td>4</td>
<td>5</td>
<td>29</td>
</tr>
<tr>
<td>Normal (Academic) students (n=16)</td>
<td>12</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>16</td>
</tr>
</tbody>
</table>

S1: Numerical; S2: Constructive; S3: rearranging the original figures; S4: viewing the original figures are viewed as part of a larger rectangle

DISCUSSION

For the Normal (Academic) students, their choices of best-help strategy did not seem to vary between the linear generalising task and the quadratic task. Their top choice was the numerical strategy S1, followed in descending order by S2 and S3. S4 was not picked by them at all. The high frequencies of these students choosing the numerical method in both generalising tasks clearly suggest that a substantial majority of them prefer to work out the functional rule using this method compared to any of the other three given figural methods. An examination of their justifications revealed that its popularity lies in its simplicity for them to represent the changes across the different cases without having to draw any diagrams, thus making the workings easier to understand. In addition, some students found that using the table
of values is a well-organised and systematic way for them to detect the pattern and derive the rule.

Unlike the Normal (Academic) students, the numerical method emerged the top choice for the Express students only for the linear generalising task but slipped to the second spot for the quadratic task. What is interesting to note about this finding is that some of these students seemed to be more aware of the applicability of this strategy to the quadratic task than their Normal (Academic) peers. They might have realised that while the numerical method shows how the pattern grows clearly in a table, the derivation of the quadratic rule is not as straightforward and easy as it appears. In fact, it is anticipated that such a method would pose a real challenge to all the Secondary One participating students. Therefore, it is not at all surprising to find some of these Express students abandoning the numerical strategy for a figural one in Christmas Party Decoration, thus causing a dip in its frequency by nearly one-half as compared to that for High Chair. As for the Normal (Academic) students who are regarded academically weaker, it is rather expected of them to not recognise the real difficulty of employing the numerical strategy to obtain the quadratic rule.

The popularity of the numerical strategy could also be traced to another plausible reason as suggested in a few students’ justifications. The students explained that the numerical method was picked as the best-help strategy because it was the only method demonstrated by their mathematics teachers. This student revelation is consistent with evidence from another recent study of ours, which showed that the majority of the participating secondary school mathematics teachers would use the numerical strategy in class to show students how to work out the rule underpinning a pattern (Chua & Hoyles, 2010b). The student revelation also highlights a precarious situation students could be facing when they are only taught, in particular, what Bezuszka and Kenney (2008) called the repeated substitution strategy and lack exposure to other types of generalising strategies. They could be misled to think that such a strategy is an effective method that can work easily for all types of generalising tasks.

Some valuable insights have also emerged from the students’ justifications of their choice of strategy. There were students who preferred the numerical method due to its clarity and simplicity, which made pattern detection and understanding easy. Subsequently, this led to the ease of obtaining a rule, a view which Bezuszka and Kenney (2008) had also pointed out. Those who eschewed this method generally found it time consuming, confusing and tedious to set up a table of values. Those who opted for figural methods reported that they were helpful in finding the rule quickly. Moreover, there was an explicit link between the size number and the number of cards used. The pattern was also easy to visualise and eventually to obtain the rule. Like those who shunned the numerical method, students who disliked the figural method lamented that it was tedious to draw and difficult to visualise the diagrams.
CONCLUSION

The present study provides a window for teachers, teacher educators as well as curriculum developers to understand which generalising strategies would facilitate student visualisation of the structure underpinning the pattern. The findings showed that the Normal (Academic) students seemed to prefer the numerical method compared to the figural method whereas the Express students tended to favour the figural method for working out the rule. Such research-based knowledge is useful to the teaching and learning of number patterns, teacher training as well as curriculum design. For instance, teachers seeking an idea of what might be an appropriate generalising strategy to employ in class when demonstrating examples can use the findings to help them make informed decisions. Aligning their choices of generalising strategies with that preferred by students can support the efficacy of teaching and learning outcomes.

Looking from another perspective, the findings of this study also draw attention to a few implications for teachers. First, teachers will need to look into the assumptions that they are making when deciding on the kind of strategies to use in class. For pattern generalisation in particular, teachers will need to be keen observers of how their students express generality to find out how they process the strategies. Second, teachers will also need to be familiar with the different generalising strategies so that they can lead students to work out the functional rule. Finally, while the findings may be preliminary since the present study is still on-going, they appear to hold promise of creating a greater awareness amongst teachers, teacher educators and curriculum developers of what students are actually capable of doing and learning. By making an attempt to understand how students visualise patterns can help teachers and teacher educators in planning more effective teaching and learning experiences, and curriculum developers in curriculum design to improve students’ ability to make generalisations.

REFERENCES


