This article presents a qualitative study on teachers’ beliefs about applying geometry, setting applied-oriented beliefs in the context of the teachers’ whole geometry curricula. Surprisingly, geometry is not perceived as a field of good applications, especially not for model building. Three classes of objections are discussed and connected to a prevalent Euclidean view on geometry and a preference for proofs and problem solving tasks. Despite these objections, applications are taught nevertheless, but the analysis of the teachers’ tasks implies that the teachers’ implicit theory of applying geometry differs from didactical requests and is not compatible with common approaches to model building. The teachers’ predominant alternative, which is called a propaedeutic use of geometry, is described in detail.

CURRICULAR ASPECTS OF TEACHERS’ BELIEFS ON GEOMETRY

The research on teachers’ beliefs has become a prospering branch of mathematics education to reveal subtle influences on students’ learning. The focus of this study lies on secondary school teachers’ beliefs on teaching geometry in the context of applied mathematics. Beliefs are customarily understood as “psychologically held understandings, premises, or propositions about the world that are thought to be true” (Philipp 2007, p. 259). In contrast to the general notion of beliefs, this study is only interested in a subset of teachers’ beliefs which has a similar content, structure, and purpose as a written curriculum. This part of a teacher’s belief system is called his individual curriculum (cf. Eichler, 2007). Its aspects – content, structure, and purpose – can be explained as follows (cf. Stein, Remillard, & Smith, 2007): The purpose of an individual curriculum is equal to the function of a written curriculum, i.e. it is used to structure lessons and to guide the instructional practice through various steps of contents and methods to goals of education. Hence, its structure can be seen in means-end relations between mathematical content, methods, and educational goals.

The teachers’ beliefs on the mathematical content are split into beliefs about concepts, theorems, objects, tasks, and textbooks. The beliefs on educational goals are separated into three levels of generality: content-specific abilities, general competencies, and top-level goals of education. The choice of the competencies enquired during the data collection is guided by the written curriculum the teachers have to act on: arguing, problem-solving, modelling, communicating, formalising, algorithmising, and using mathematical description and symbols (KMK 2004, p. 7).

Against this background, a qualitative study was designed to examine the individual curricula of nine teachers on teaching geometry at German higher-level
secondary schools (so-called Gymnasien, in which about 35-40% of the German students are taught and whose school leaving certificates are normally necessary to get access to university). To invite teachers as participants of this study, four districts of the governmental school hierarchy in different regions of Germany were contacted, each responding by a list of two or three teachers willing to participate in this study. The teachers were visited and interviewed by the author.

The focus of the curricular aspects was chosen, since investigations on the implementation of curricula are scarce in Germany; and in addition, no study has been carried out on the issue how the changes of the new national curriculum (KMK 2004) are reflected by teachers – especially its emphasis on the competencies mentioned above and its fortified accent on applications and model building. This lack of information was the reason to focus this study on modelling and to examine how beliefs on this topic are integrated into the teachers’ whole geometry curricula.

Individual curricula as subjective theories can provide a bidirectional contribution to mathematics education (cf. Girnat 2010): It is possible to detect disparities between prescribed goals and the teachers’ objectives to react on possible maladjustments in practice. On contrary, individual curricula can also be analysed in comparison with didactical opinions on a cooperative level to integrate teachers as semi-professional researchers and to expedite theoretical thinking on the ground of differing views from classroom practice. Both aspects are pursued within this study.

THEORETICAL BACKGROUND, METHODS, SETTINGS, AND DATA

The data were collected by semi-structured in-depth interviews, each taking about 90 minutes. They were interpreted according to the research programme of subjective theories (Groeben et al. 1988). This framework was invented by psychologists to collect and interpret complex systems of beliefs used by professionals to make their decisions when acting occupationally on the basis on a more or less commonly shared, but individually interpreted theory, containing empirical knowledge and normative prescriptions similar to curricula or didactical theories. Due to the usual complexity of a professional’s subjective theory, a qualitative approach is normally preferred and also used in this study.

To interpret the data, a so-called dialogue-hermeneutic method was invented (Scheele & Groeben, 1984), consisting of three steps: an interview to collect the main data, an interpretation of the data by hermeneutic methods to define the subjective theory, and a spot check observation of the participants’ behaviour to validate if the assumed subjective theories are in fact relevant to the teachers’ acting. In this case, the observation consisted of five lessons per teacher and a collection of the applied-oriented tasks used in about the last quarter before the observation.

The teachers’ geometry curricula were analysed in all the aspects mentioned above, and not limited to topics of applications, mathematisation or modelling. The reason for such a “holistic” approach is the idea that the common instructional practice is guided by several goals of education not necessarily related to applications. Hence,
the applied-oriented goals have to find their places within the totality of curricular aims and convictions. The central questions of this study can be only answered concerning a whole individual curriculum: What significance do the teachers attribute to applied-oriented goals? Of what kind are the connections from applied-oriented goals to other goals of education? Are the teachers’ applied-oriented goals similar to or different from didactical ideas and the new written curriculum?

THE FOCUS ON MODEL BUILDING

Before presenting some results, it is necessary to sketch some didactical perspectives on applied mathematics briefly. The most essential issue seems to be the relationship between general mathematical theories and empirical knowledge on singular situations. Kaiser-Meßmer (1986, pp. 83-92) has proposed a classification whose extremities are called the pragmatic and the scientific-humanistic approach: The latter emphasizes mathematical concepts and theories as the main goals of education and incorporates real-world situations mainly as subordinate tools to develop mathematical concepts and insights on manifold realistic associations. Empirical knowledge is of minor interest; and the teaching process follows a mathematical taxonomy of problems, concepts, and techniques and is not derived from empirical questions connected to real-world situations. The real-world situations are just “illustrations”. The pragmatic view, on contrary, stresses the empirical insights on real-world situations and includes a meta-theory about the relationship between mathematics and reality to be picked out as a central subject when teaching applied mathematics. This approach rests on three classes of educational goals (Kaiser-Meßmer 1986, p. 86): 1) Utilitarian aims: The situations are not selected according to mathematical taxonomies, but on the basis of the current or expectable benefit to the students’ lives. 2) Methodological aims: The students shall obtain general competencies and meta-knowledge on applying mathematics. 3) Meta-scientific aims: Applying mathematics is perceived as model building. The concept of model building can be explained by the model building cycle and has to be reflected in classroom practice as “one of the main components of the theory for teaching and learning mathematical modelling” (Kaiser, Blomhøj, & Sriraman, 2006, p. 82).

Both approaches of applying mathematics imply different standpoints on goals of education in general: The pragmatic view sees its contribution in universal model building competencies and a preparation to life situations. The scientific-humanistic approach instead rests on the generality of mathematical theories. The new written curricula in Germany based on the national prescriptions (KMK 2004) underline the pragmatic view and introduce model building as to be obligatory, which was facultative or even not mentioned in former German curricula. For this study, a simple version of the manifold modelling cycles (fig. 1) is used, which seems to be sufficient to determine if a teacher possesses a concept of model building in the sense of the contemporary academic debate and written curriculum.
FINDINGS: THREE OBJECTIONS TO GEOMETRICAL MODELLING

To summarise the results, seven of the nine teachers have got objections to integrate applications into their lessons on geometry, though being open-minded to modelling in other parts of school mathematics. The spectrum of objections ranges from a strict exclusion to a moderate use. But even if geometrical applications are taught, the way of applying geometry differs from didactical suggestions. The interpretation of the data leads to three classes of objections which are based on different reasons.

Ontological Aspects

The strictest opposition against an applied-oriented way of teaching geometry is based on ontological beliefs about the nature of geometry and its objects. In Girnat (2009), the following classification of geometrical ontologies is proposed (fig. 2).

![Ontological classification of geometry](image-url)

**Figure 2: An ontological classification of geometry**
This classification rests on two aspects: 1) The theoretical aspect: Is geometry taught on the basis of a given axiomatic Euclidean theory (whose rigour may be restricted to an adequate school level) or is it derived from experience, observations, and measurement as an empirical theory? 2) Intended application: Does geometry refer to idealistic objects in the sense of Plato or to physical objects or is it regarded as purely formalistic in the sense of Hilbert?

The two aspects of this classification follow the main ideas of the theory of geometrical working spaces (cf. Houdement & Kuzniak, 2001). According to this approach, geometry is split into three paradigms: A formalistic theory (called G3), an idealistic theory strictly based on deductive arguments (G2), and an empirical theory based on measurement and experiments (G1). Compared to the classification used here, the theory of geometrical working spaces combines the aspects of justification (deductive or empirical) and reference (the connection to objects) to each others, leading to the consequence that an empirical reference is tacitly bounded to empirical, non-deductive methods in G1. For our purposes, it is necessary to separate these aspects and to allow a geometry which refers to empirical objects, but is mainly based on deductive arguments (just including some empirical initial conditions). This type of geometry is called the rationalistic one.

**Idealistic Platonists: No applications intended**

Two of the teachers can be classified as exponents of an idealistic view on geometry: They do not perceive geometry as a theory of real objects, but of ideal entities which correspond to ruler-and-compass constructions and which fulfil the theorems of the Euclidean geometry without any aberrations.

Mrs. D: The beauty of mathematics is the fact that everything is logical and dignified. […] Everywhere else, there are approximations, but not in mathematics. There is everything in this status it has ideally to be in. [It is important for the students] to recognise that there are ideal things and objects in mathematics and that, in reality, they are similar, but not equal.

From this point of view, applying geometry is barred by definition. Instead, constructive descriptions are promoted to get access to ideal objects. Physical objects, typically limited to drawings, are only used as symbolic representations of the “true” ideal objects of geometry. Every empirical investigation is seen as a heuristic tool, but does not have any relevance for justifying geometrical insights.

Mrs. D: Besides proof abilities, problem solving is in fact the most important thing I want to convey in my lessons on geometry.

Mr. C: If geometry just consisted of measuring, calculations, drawing, constructing, and land surveying, then I would regard it as poor. […] [Geometry as a] tool to get access to the real world? No, problem solving would be my favourite. Why? Problem solving is a keyword that includes everything. It is the final goal to make students work systematically, identifying premises and drawing conclusions to solve a problem.
The classroom observations support the impression derived from the interviews: The lessons on geometry held by these two teachers are focussed on proofs, constructions, and problem solving task, using drawings only as heuristic tools.

**Model building versus proof and problem solving tasks**

Not only for the two “idealistic” teachers, proof and problem solving tasks are seen as the main issue of teaching geometry; but even for six other teachers who are not strict opponents against geometrical applications, applying mathematics is not a top level goal, but rather appears as subordinated to proof and problem solving competencies. The tasks the teachers presented as good examples to convey these competencies match the typical characteristics of problem solving tasks (Holland 2007, pp. 170-195) and lead to the hypothesis that aspects and methods demanded there are in contrary to the settings of a model building process. The main differences are summarised as follows (tab. 1):

<table>
<thead>
<tr>
<th>Aspect</th>
<th>Model building</th>
<th>Proving or problem solving task</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Objects of interest</strong></td>
<td>singular situation</td>
<td>general theorem or configuration</td>
</tr>
<tr>
<td><strong>Access to objects</strong></td>
<td>by measurement and experience</td>
<td>by constructive descriptions</td>
</tr>
<tr>
<td><strong>Building a real model</strong></td>
<td>by simplifying</td>
<td>simplifying not allowed</td>
</tr>
<tr>
<td><strong>Mathematical treatment</strong></td>
<td>inventing a mathematical model</td>
<td>using known operators, theorems, or methods</td>
</tr>
<tr>
<td><strong>Validation</strong></td>
<td>empirically</td>
<td>by deductive arguments</td>
</tr>
</tbody>
</table>

**Table 1: Differences between model building and proving or problem solving tasks**

Most of the teachers want to prevent their students from getting confused by mixing the standards of modelling, proving, and problem solving. So, they typically split their courses on geometry in pure and applied-oriented sections:

**Mr. B:** Geometry as a tool to get access to the real world is legitimately not in the first place. An application is useful to introduce a new subject, to legitimise it, and to test the competencies of this field by realistic tasks in the end. But in between, a lot has to be done without any reference to the real world, detached from these accessory parts which are not important to the mathematical theory. In between, applications are counterproductive. They seduce the students not to argue strictly deductively.

Insofar, the deductive view on geometry is predominant and geometrical applications are mostly seen as appendices to the “serious” treatment of geometry, which leads to the consequence that aspects of model building are not integrated into the geometry curriculum and that geometry is “applied” to real-world problems in the majority of
cases in the way that a problem-solving task is “decorated” by an empirical sounding vocabulary, which is seen as characteristic for a rationalistic use of geometry.

**The contrast: Geometrical applications as the main focus**

To contrast the first two types of objections, the only teacher who approves and practise applied-oriented tasks extensively shall be quoted comprehensively. After arguing for teaching “geometrical modelling”, Mr. H was confronted with the question which significance proofs and problem solving tasks had in his opinion:

Mr. H: Proofs, not in the sense of what is called a proof at university, shall demonstrate that something could be plausible, more plausible than something different. […] My students shall be able to judge if a solution can be plausible, if the units match, if something could really have happened […] if you throw a stone into a basin of water, then if the gauge could really rise by 3 meters. […] They have to solve specific problems, and they have to use their geometrical tools. That needn’t to be exact, that depends on the situation, and they have only to know if the methods and solutions may be plausible and realistic and how they can be used in the specific context.

Mr. H stresses the pragmatic aspects of applied mathematics, holding an empirical view on geometry on G1 level; and in doing so, he dispenses the educational goals which the teachers mentioned above pursue by proofs and problem solving.

**Geometrical applications as being uninteresting**

The third and last class of objections against geometrical applications are based on the assertion that they do not lead to interesting insights.

Mr. A: The better applications can be found in algebra or stochastics, per cent calculations, linear optimisation. It is important to get a deeper insight into reality by modelling. In geometry, there are such things as dividing a pizza by a compass. I saw a trainee teacher do so. That’s ridiculous.

It is remarkable that this objection is focussed on purely geometrical applications. The classroom observations and the teachers’ statements on “good” examples for modelling reveal that the teachers in fact use geometrical applications, but that they are small and, for themselves, uninteresting parts of more complex modelling tasks guided by non-geometrical questions. These tasks typically possess a two-step structure: In the first step, geometry is used to calculate some initial or boundary conditions, e. g. some lengths, areas, or volumes. Afterwards, these values are committed to a second, non-geometrical step which includes stochastics, algebra, optimisations or a problem derived from the natural or social sciences, e. g. some price, weight or velocity calculations. Five teachers used these two-step applications and stated that the interesting insights first and foremost arise in the second step. A typical example is mentioned and explained by Mr. B:

Mr. B: To grasp the sense of what I like to say, let us regard the following task: “A businessman wants to sell salt in small rectangular packages of 250 gram.
What would be your advice to reduce the waste of material?” That’s an interesting problem providing some surprise, if you take the situation serious, and it is quite challenging, but the geometry in it is not, it’s standard, it’s only a vehicle to manage the interesting aspects, and it has to be well understood before deliberating about this problem.

If the modelling cycle is the core concept to analyse the learning and teaching of applied mathematics, then it will be difficult to reconstruct this two-step applications by a cyclic structure (fig. 1). It rather seems adequate to perceive geometry as “propaedeutic” to modelling, outside and “a priori” to the modelling cycle (fig. 3).

![Figure 3: Geometry as propaedeutic to model building](image)

The meaning of “propaedeutic” can be explained in three aspects: 1) A propaedeutic use of geometry is characterised by a static view on geometry: It is seen as a pre-established theory, based on rigid concepts, proved theorems, and infallible methods. 2) A propaedeutic geometry is used as a suitable language and reliable background theory to structure and simplify a situation by geometrical concept. Insofar, this way of applying geometry is different from a modelling process, since geometrical concepts are already used to structure the real situation, and not to build a mathematical model after structuring the real situation independently. Hence, the use of geometrical concepts and methods is prior to any kind of mathematisation in the sense of the modelling cycle. This aspect is best to observe in the two-step structure of the teachers’ “good” examples for modelling tasks: Geometrical concepts and theorems are already necessary to “see” geometrical objects in reality and to calculate the values of areas or volumes before the second step, the “true” modelling process, can get started. Additionally and as a further contrast to the modelling cycle, the propaedeutic use of geometry does not include any kind of validation, since geometrical theorems and methods are used as already proved. In the salt example, the relevant second step is the optimisation process, based on proposals how to shape
the packages. The calculation of its shells, volumes, and cut-offs is just an algorithmic task, based on pre-established geometrical knowledge and methods.

3) The teaching method is propaedeutic, since most of the teachers follow Mr. B’s suggestion to avoid connections to reality at first and to integrate realistic situations in the end of a teaching unit. The observations indicate that five of our teachers approve modelling tasks and pose them in their lessons, but either geometrical problems are not involved and are taught separately or geometry is integrated propaedeutically into a two-step structure.

CONCLUSIONS

The study reveals objections against an applied-oriented approach to geometry based on three reasons: a traditional idealistic view on geometry, a preference for proof and problem solving competencies, and a propaedeutic treatment of geometry. Especially the pragmatic view on modelling and the model building cycle as its core concept could not be found as a part of the teachers’ geometry curricula and teaching practice, though being observable in non-geometrical contexts.

It is interesting to see how several of the teachers integrate applied-oriented aspects to their geometry curriculum on basis of their “non-applied-oriented” view, combining a Euclidean perception of geometry with a propaedeutic use. This finding suggest some further reflections: Although the classroom observations of most teachers reveal no applied-oriented tasks which could be described as “good” modelling tasks in the sense of the academic debate and the new German curriculum, the teachers are not just unwilling to teach geometry applied-oriented, but are focussed on educational goals connected to proving and problem solving tasks, which are parts of the written curriculum, as well, and which presuppose a geometrical ontology and methodology that provokes a conflict with the background theory of modelling objectively, and not only in the subjective perceptions of these teachers. The academic debate has to state a proposal how to manage these conflicting demands in practise.

Especially the observed two-step structure poses an interesting question to the academic debate on modelling: Is this way of teaching applied geometry just a consequence of the teachers’ traditional Euclidean view on geometry or is it based on a typical way how geometry “naturally” refers to reality? In the latter case, it would be questionable if the model building cycle is an adequate representation of applying geometry. In contrary to common didactical debates (Kaiser et al., 2006), the findings suggests the conjecture that it may be advisable to shape the modelling debate less as a “top-down theory”, establishing a single framework to be applied in every part of school mathematics identically, but more as a “bottom-up research programme”, exploring the existing uses of applications in different contexts and parts of school mathematics following the question if there are different ways how mathematics and the different parts or disciplines of school mathematics refer to reality besides the modelling cycle.
REFERENCES


