By a teaching experiment, we introduced isometries and their practice in a experimental sample of 3rd graders (8 year old), assuming that learning and training of isometries might affect the standard geometrical understanding in primary school. Comparison with a control sample of the same grade supports our hypothesis.

Keywords: Teaching experiment, Isometries, Primary School

THE ITALIAN PRIMARY SCHOOL SCENARIO ABOUT ISOMETRIES

In one learning target Italian primary school new ‘curriculum’, isometries are quoted: «To recognize rotated, translated, reflected figures.» (MPI, 2007). This topic must be treated in the two last school years, only by observation and recognition. Practice with these concepts has not a suitable development. The same curriculum states the introduction of geometry for grade 3 (but often presented in grade 4 and 5) as «To recognize, to denominate and to describe geometrical shapes.» and also «To measure segments by the means of both metre or arbitrary units; to connect the measure practice to the knowledge regarding numbers and operations.». This practice fosters a nominalistic approach.

THEORETICAL FRAMEWORK

Felix Klein (1890) reorganized geometry suggesting that the ‘content’ of its founding is the concept of group of transformations. These subjects run into difficulties in Italian school. Therefore the topic is often assumed as unrelated with the ‘true’ geometry (Iaderosa & Malara, 1998); this remark is still worth. It is perceived as a ‘different kind’ of geometry for which teacher has neither traditions nor standards for the teaching and for the assessment, hence it is relegated to an occasional practice.


Our teaching experiment in grade 3 proposes an innovation for bettering geometry learning, starting from Swoboda’s outcomes. We assume that enhancing the learning of isometries we will improve the standard geometry learning. With our approach we want to develop ‘flexibility’, i.e. the use of a variety of strategies and/or the skill of adaptive strategy choice to task specific characteristics, a resource for mastering the everyday life problems. In fact, isometries are a worth training since they ask to mas-
ter simultaneously static and dynamic aspects. The experiment uses artefacts which are concrete pieces of paper. They ask a continuous passage between different cognitive levels: they ‘call on’ geometric figures by the means of the drawings on them, taking in account its idiosyncratic features. The same tile can be ‘read’ differently, depending on pupil’s attention (Marchini et al., 2009).

In the 3rd stage through Escher’s drawing as artefacts we verify if the practice with isometries on simple drawings can be transposed to complex non-standard shapes.

THE EXPERIMENT: Aims and planning

Experiment aims are: 1st aim - Are isometries a suitable topic for grade 3 pupils? 2nd aim - Do plane isometries learning affect the (above) ‘standard’ Italian school geometry? Positive answers to these questions can support our proposal of the innovative introduction (1st aim); isometries could play a relevant role for integrating deeply the traditional teacher’s practice in geometry with transformations (2nd aim).

We planned: an experimental sample (ES) (40 learners), a control sample (CS) (39 pupils), a pre-test (PT) in both samples, a treatment in ES, and the final test (FT) in both samples, one school year later [2]. Treatment and tests assessment are researcher’s duty; ES teachers record the treatment sessions, assure the discipline, and administer the tests. We ask ES and CS teachers to continue her/his projected teaching, without reference to the PT. In particular, in the second school year, ES teachers must avoid reference to isometries.

THE EXPERIMENT: Pre-Test

For obtaining a ‘portrait’ of the two samples, the experiment started (2008/2009) with PT for ES and CS. Test consisted in three sheets, here named Shepherds, Pizza and Patterns [3].

<table>
<thead>
<tr>
<th>Sample rates</th>
<th>Shepherds</th>
<th>Pizza</th>
<th>Patterns</th>
<th>PT Tot</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sh1</td>
<td>Sh2</td>
<td>Sh3</td>
<td>Sh4</td>
</tr>
<tr>
<td>ES</td>
<td>45</td>
<td>82</td>
<td>32</td>
<td>24</td>
</tr>
<tr>
<td>CS</td>
<td>12</td>
<td>88</td>
<td>9</td>
<td>24</td>
</tr>
<tr>
<td>χ²-Test</td>
<td>0.21</td>
<td>43</td>
<td>1.76</td>
<td>99</td>
</tr>
</tbody>
</table>

Table 1. Results of the PT: rate of exact answers and χ²-test probability

Issues employ everyday language, avoid mathematical terms, require visual estimate regarding perimeters and area and isometries, but neither a geometric formal knowledge, nor a specific teaching. We assessed the answers as Yes – Not.

<table>
<thead>
<tr>
<th>Sample</th>
<th>No.</th>
<th>Shepherds</th>
<th>Pizza</th>
<th>Patterns</th>
<th>PT</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Aver. score</td>
<td>σ</td>
<td>z</td>
<td>Aver. score</td>
</tr>
<tr>
<td>ES</td>
<td>38</td>
<td>1.82</td>
<td>1.05</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CS</td>
<td>33</td>
<td>1.33</td>
<td>0.53</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2. Results of the PT: statistical test about average scores
Table 1 resumes the rate of success, and the related probability of \( \chi^2 \)-Test for the statistical relevance of the score differences between samples [4]. Table 2 aggregates results of Shepherds’, Pizza and Patterns questions, with the statistic ‘\( z \)’. The global ‘portraits’ of the two samples have non-relevant differences. In PT only difference in patterns issues is favourable to CS children and it is statistically probable. It can be surprising since the right solutions require a ‘sensibility’ to isometries such as intuition of translation for \( Pa_1 \) and of rotations for tasks \( Pa_2 \) and \( Pa_4 \). This ‘feeling’ could be useful for solving tasks \( Pz_1 \) (translation/symmetry), \( Sh_1 \) and \( Sh_3 \) (rotation), but the results of these issues show a superiority of ES children statistically relevant.

**THE EXPERIMENT: Treatment for the experimental sample**

The treatment was planned for six sessions of two hours each, one session per week, in three stages, with non-quantitative approach. Its realization lasted 16 hours.

![Figure 1. Treatment protocols in the order of their presentation to ES classrooms](image)

**The 1st stage** (6 hours). In sessions 1 and 2, ES pupils discussed in groups the first four documents of Figure 1; for each one they recorded the worth aspects, they assessed protocol, and they presented orally their ‘conclusions’. Afterwards, for each protocol Carlo made an ‘institutionalisation’ activity, drawing on the black board four squares as in Figure 2, reproducing by a schematic way the four consecutive tiles as they appear in the upper left corner of the document. Carlo asked whether there was a ‘tie-in’ between two consecutive tiles, for recognizing rotated, translated, reflected figures (MPI, 2007). Pupils accepted without problems that ‘consecutive’ meant ‘on the right’ or ‘down’. For 2A15, in a class the word *trasloco* (move) came out immediately. In the other class the idea of movement required more time. For 1A16, in both ES classes, the word *specchio* (mirror) came out without difficulty, looking at horizontal tiles disposition. Hence, during the 1st session the presence of translation (\( T \)) and (axial) symmetry (\( S \)) in the construction of protocols were detected. Pupils realized that \( T \) works in 2A15 both from left to right and from up to down, in 1A16, \( S \) acts from left to right and \( T \) going down.

During 2nd session, pupils analysed the 3rd and the 4th protocols. In the ‘institutionalisation’ phase, they found that 1A17 is the result of application of \( S \) in horizontal and in vertical. We accepted the statement as a first approach in Van Hiele’s (1986) level 1. Pupils’ analyses of 2A16 agreed that its author made a ‘mistake’. The protocol was chosen on purpose, for making evident the presence of a rule in the construction of the protocol, by the means of its violation. The different appearance of protocols 1A17 and 2A16 could hide the fact that the construction rule is the same. In the ‘institutionalisation’ phase this identity came out. In Vigatto school pupils suggested
that the mistake was the fact that a tile was turned. Carlo seized the opportunity to introduce rotation (R - *rotazione*), avoiding the discussion of Co10. During 3rd session, in Vicofertile school, the discussion of Co10 allowed the introduction of rotation.

The 2nd stage (4 hours). It aims at the consideration of isometries as mathematical objects. We prepared a card game: three ‘playing’ cards with the letters T, S and R in a urn, and the game board of Figure 2 as an array of two times two squares (on the blackboard). The four squares determine a cross (it is grey in Figure 2). Now the rule is to draw, with restitution, four times one card at time and to write the letter of that card in the empty cross arms, in this conventional order: top - left - right - bottom. Finally a tile is placed (or drawn) in the top left board square. Pupils copied the stuff on their exercise-book. E.g. with a suitable choice of the tile and its placement we get: T, T, T, T, for 2A15; S, T, T, S for 1A16; S, S, S, S for 1A17.

We gave a homework task with the sequence of letters R, S, R and T, and a tile of the type used in 2A16. Children proved flexibility by concluding task impossibility. During the next homework discussion, pupils suggested possible different rotations of multiples of ‘one quarter clockwise rotation’. Therefore the cards R1, R2 and R3 were added in the urn instead of R. The playing with the new card game concluded the 2nd stage. In this way pupils produced protocols on the basis of simple [5] rules (Marchini & Vighi, 2011) showing a good mastery of isometries. To single out simple instances of impossible tasks is an effective way to introduce pupils to control procedures. The card game with isometries could be used in every school environment for to raise isometries, both as procedure and mathematical objects. In our experiment we avoided ‘structural’ properties of functional composition, but these topics can be useful in other grades.

The 3rd stage (6 hours). Paola recalled plane isometries through Escher’s paintings: Escher’s 28 (shortly E28) for translations; E79 for rotations and E12 for symmetry. E55 presents shapes suitable as a summary of previous plane isometries. The aim was to attach attractive and affective aspects to transformations. For each drawing Paola asked children, in sequence, “What can you see in this drawing?”; then to individuate with letters or colours many figures obtained from a starting figure by a suitable isometry. In particular, for rotation, she required to individuate ‘rotation centres’. The shape complexity hampered only few pupils.

THE FINAL TEST: Analysis of FT

We choose, on purpose, a test which is far enough (in time and topics) from the teaching of isometries. The tasks can be considered suitable for children having a standard teaching of geometry; nevertheless the issues require a geometrical thinking since straightforward applications of rules for perimeter and area are not enough.
The FT (in Enclosure) was administered in 50 minutes at the end of grade 4 in school year 2009/2010. It was inspired to some PT items. The issues of the test were problematic for children, since they learnt perimeters of rectangles and a few about area. The leading idea was to assign problems about three roughly ‘rectangular’ figures in which we gave the measure of the length for some segments (represented in proportion); for solving them the application of isometries can be useful or necessary. Some data are missing; they can be found by geometrical thinking and arithmetic computation (with an implicit didactic contract suggesting that what looks like a rectangular shape is a rectangle, or congruent-like parts of the same shape, are congruent). In our opinion, the identifying the missing data requires a sort of deduction in a ‘natural axiomatic’.

For perimeter of Shape 1, six data are given and two are missing; for perimeter of Shape 2, eight data are given and four are missing [6]. Therefore computation of perimeter involve long addition with decimals. In the first case it is necessary to solve the equation \(x + y = 5.0 + 1.5\); in the second case, solution is found by the means of the equations \(6.5 = 3.0 + x + 1.5\) and \(1.0 + 2.0 + y = 4.5\). The drawings help to avoid algebraic computations since evidence suggests solutions with the help of a deduction in a ‘natural axiomatic’.

For Area 1, it could be useful to add 5.0+1.5, and then to make \(4.5 \times 6.5\). Otherwise the shape can be divided in two rectangles. For Area 2 and Area 3, all the necessary data are given, and search of missing data can only corroborate the hypotheses of congruence of some pairs of pieces from the same shape. The difficulty of computing Area 3 can be solved correctly in two different ways.

a) The first way: the solution can be found only by insight (Divišová & Stehliková, 2010) i.e. by an intuition of congruence, similar to the one required in items of PT, since in Shape 3 there is a part in form of semicircle for which 4th graders do not know an appropriate formula for area. This way requires only the computation of \(4.5 \times 6.5\). Thence pupil can return back to Area 1 and Area 2, recognizing a local isometry [7] which can be applied to small rectangles.

b) The simplest way of computing areas Area 1 and Area 2 is to recognize that all the shapes become equal to a rectangle with side length 4.5 cm and 6.5 cm, by suitable shape decomposition. The congruence of suitable parts of each shape can be proved by appealing to local plane isometries. Therefore the direct computation of Area 1 and Area 2 could be considered a sort of ‘distractor’, even if it, with the sameness of results, can give the good hint for Area 3, by recognizing the role of local isometries to semicircles.

An analysis from different points of view is in (Vighi & Marchini, 2011).

THE FINAL TEST: Results of FT

We assessed FT protocols in various ways. The simplest is to assign the score 1 for the correct numerical value and 0 for the wrong or missing numerical value, as a
measure of the ‘understanding’ (Kilkpatrick, 2009), connected with the result. Another kind of data is the average number of children which try to solve the problems. We consider the essay, in itself, a positive behaviour towards the topics, so we label it as ‘confidence’ [8]. Lastly we look at solving procedures, disregarding possible mistakes, the ‘competence’ (Godino, 2003). We considered a right procedure the product $4.5 \times 6.5$, or for Area_1 and Area_2, a suitable shape decomposition in rectangles with the corresponding products. As to perimeters we considered a right procedure when all the missing data are found and summed to the given one, or even if one of them was forgotten, for scarce attention, but the finding of the other missing data is a sufficient proof of competence.

### Table 3: Results of the FT: Average values and statistical tests

<table>
<thead>
<tr>
<th></th>
<th>Perim_1</th>
<th>Area_1</th>
<th>Perim_2</th>
<th>Area_2</th>
<th>Area_3</th>
<th>Perimeters</th>
<th>Areas</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>ES score</td>
<td>0.18</td>
<td>0.13</td>
<td>0.11</td>
<td>0.24</td>
<td>0.18</td>
<td>0.29</td>
<td>0.55</td>
<td>0.84</td>
</tr>
<tr>
<td>CS score</td>
<td>0.27</td>
<td>0.06</td>
<td>0.03</td>
<td>0.09</td>
<td>0.03</td>
<td>0.30</td>
<td>0.18</td>
<td>0.48</td>
</tr>
<tr>
<td>Statistical test</td>
<td>$\chi^2=37$</td>
<td>$\chi^2=32$</td>
<td>$\chi^2=22$</td>
<td>$\chi^2=10$</td>
<td>$\chi^2=4.08$</td>
<td>$z=-0.10$</td>
<td>$z=1.91$</td>
<td>$z=1.34$</td>
</tr>
<tr>
<td>ES confidence</td>
<td>0.82</td>
<td>0.79</td>
<td>0.82</td>
<td>0.76</td>
<td>0.76</td>
<td>1.63</td>
<td>2.32</td>
<td>3.95</td>
</tr>
<tr>
<td>Statistical test</td>
<td>$\chi^2=12$</td>
<td>$\chi^2=24$</td>
<td>$\chi^2=7.1$</td>
<td>$\chi^2=0.16$</td>
<td>$\chi^2=0.07$</td>
<td>$z=-0.13$</td>
<td>$z=3.39$</td>
<td>$z=2.06$</td>
</tr>
<tr>
<td>ES competence</td>
<td>0.32</td>
<td>0.29</td>
<td>0.26</td>
<td>0.39</td>
<td>0.42</td>
<td>0.58</td>
<td>1.11</td>
<td>1.68</td>
</tr>
<tr>
<td>Statistical test</td>
<td>$\chi^2=49$</td>
<td>$\chi^2=8$</td>
<td>$\chi^2=84$</td>
<td>$\chi^2=0.94$</td>
<td>$\chi^2=0.05$</td>
<td>$z=-0.14$</td>
<td>$z=3.35$</td>
<td>$z=2.32$</td>
</tr>
</tbody>
</table>

The low average score in Table 3 results of many mistakes in computations and/or in procedures. We aggregate the scores for perimeters, areas and global. Without another table of quantitative data, we can conclude by ‘$z$’ statistic that differences between the scores of the two samples are not statistically relevant.

Table 3 shows the great difference between confidence, competence and understanding of the topics, using this names in our meaning. CS children are more confident about perimeter, but difference is not statistically relevant. Moreover ES children show a greater confidence with area than the CS children, and this difference is statistically significant. The $\chi^2$-test states that this fact is due to the confidence with Area_2 and Area_3. With data aggregation (perimeter+area), samples difference in confidence is statistically probable only for Shape_2; difference in competence for Area_2 and for Area_3 are statistically significant.

### THE FINAL TEST: Analysis and interpretation of the FT results

In school year 2009/2010, in both samples, measure of length, perimeters for triangles and rectangles were taught and area was introduced briefly. CS teacher treated also isometries. This substantial knowledge ‘equivalence’ of the two samples is stated by the global FT results of Table 3 for Shape_1, which is closer to standards.

ES confidence presents similar results for perimeter and area. For CS, instead, the differences about perimeters are statistically significant, both for confidence and competence. Moreover there are statistically probable differences for confidence with Area_1 and Area_2, and for Area_1 and Area_3.
The greater number of missing data could be an obstacle for determining Perim$_2$: from Table 3 it seems that the reductions in average scores should be provoked by mistakes in the sum because of the number of addends. Only one protocol seems to determine smartly Perim$_2$ by adding 1 cm (or 0.5 cm twice) to Perim$_1$.

Few protocols explicit procedure. We got 7 protocols (6 of them in ES) in which all the three problems with area pupils obtained the same results (even if wrong). As regards scores, the number of right computations lowers: we detect 4 cases, 1 of them in CS. Equality of Area$_2$ and Area$_3$ only is affirmed by 3 ES pupils and 1 CS child.

An ‘isometric thinking’ can be present in 10 ES pupils since at least one perimeter is computed wrongly by the sum $4.5 + 6.5 + 4.5 + 6.5$. In fact, 5 of them applied at least once to correct procedure for area (4 of them for both areas). They look unaware that local isometry preserves area, but does not keep perimeters.

In Area$_2$ and Area$_3$ are involved local roto-translations; for Area$_1$, instead, we can consider local translation or local axial symmetry (as stated explicitly by one CS pupil). It could be relevant the fact that the application of procedure identifying Area$_2$ and Area$_3$ is the most frequent (10 ES, 1 CS) and when a pupil individuate the equality of Area$_1$ (the simplest) and Area$_3$ (the most difficult), then s/he possibly comes back for obtaining the same result for all the shapes. On the basis of the previous remarks, of Table 3, and by the fact that child can determine Area$_3$ only by intuition of a local roto-translation, we can state that in CS a method based on local translation / symmetry is applied more than the one requiring roto-translation. The ‘equality’ of average confidence for Areas in ES could be justified with the previous learning of isometries.

**COMPARISON BETWEEN PT AND FT**

We can compare the results of FT and PT taking in account the ‘sameness’ of intuition/knowledge necessary for solving the tasks.

<table>
<thead>
<tr>
<th>Average no. children</th>
<th>Perim$_1$ &amp; Perim$_2$/Sh$_4$</th>
<th>Area$_1$/Pz$_1$ &amp; Pa$_1$</th>
<th>Area$_2$ / Area$_3$/Sh$_1$, Sh$_3$, Pa$_2$ &amp; Pa$_3$</th>
<th>FT/PT</th>
</tr>
</thead>
<tbody>
<tr>
<td>ES improve</td>
<td>0.66</td>
<td>0.29</td>
<td>0.42</td>
<td>0.11</td>
</tr>
<tr>
<td>ES equal</td>
<td>0.29</td>
<td>0.53</td>
<td>0.45</td>
<td>0.32</td>
</tr>
<tr>
<td>ES worsen</td>
<td>0.05</td>
<td>0.18</td>
<td>0.13</td>
<td>0.58</td>
</tr>
<tr>
<td>ES balance</td>
<td>0.61</td>
<td>0.11</td>
<td>0.29</td>
<td>-0.47</td>
</tr>
<tr>
<td>CS improve</td>
<td>0.70</td>
<td>0.33</td>
<td>0.64</td>
<td>0.12</td>
</tr>
<tr>
<td>CS equal</td>
<td>0.24</td>
<td>0.45</td>
<td>0.09</td>
<td>0.21</td>
</tr>
<tr>
<td>CS worsen</td>
<td>0.06</td>
<td>0.21</td>
<td>0.27</td>
<td>0.67</td>
</tr>
<tr>
<td>CS balance</td>
<td>0.64</td>
<td>0.12</td>
<td>0.36</td>
<td>0.06</td>
</tr>
</tbody>
</table>

**Table 4. Comparison of FT and PT: Average number of change in performance**

The relevant differences, mainly for area questions, between the two samples in FT can follow from an evident initial difference in PT results as regards to problems from which FT issues were inspired. Notice that in the PT we did not ask quantitative results, therefore we think as unsuitable to compare PT with the FT scores. The influence of the qualitative treatment can be assessed with children’s confidence and com-
petence. Thence we compare the PT and FT looking at the average number of children who improved (equalled, made worse) their performance from PT to FT [9]. The last two columns are obtained by aggregation of all FT task results the PT task results.

Negative sign in Table 4 is a warning: the ‘intuitive’ test gave better result than the quantitative task. This fact could be a consequence of a few attention to a practice promoting the evolution of child’s idea towards a more complete knowledge.

CONCLUSION AND DISCUSSION

The research has clear aims, but during its implementation we faced other issues:

a) Does practice with non-conventional shapes help pupils in FT tasks?
b) Does the treatment improve flexibility?
c) Does the treatment improve pupils’ performance in FT tasks?

Research 1st aim had a big number of corroborations, during the treatment (Marchini & Vighi, 2011) and also with the permanence of the taught concepts one year later e.g. by words or drawings mention of isometries in the 50% of ES pupils. Only 5 CS pupils prove their acquaintance with isometries; they testify that the same arguments were introduced in their CS class and, by results comparison, the relevance of treatment in ES. Therefore the 2nd aim of our research has been achieved.

Area problems distinguish the most (Table 3) the confidence difference between samples. The ‘distance’ of our shapes from school practice can be measured by the average confidence, which values diminish from Shape1 to Shape3 for CS. The same values are nearly constant for ES pupils which show familiarity with complex shapes. Perimeter and area of FT shapes cannot be found by application of ‘one’ rule. They could block diligent pupils able to solve standard exercises. The FT tasks require insight (Divišová & Stehliková, 2010) or more flexibility and an inventory of geometrical tools going far beyond of the simple formulas for rectangles. Flexibility is also has been helped by a practice with isometries and complex and non-regular shapes. It is worth the improvement of competence of ES in comparison with CS for area questions (Table 3). Therefore issues a) and b) have positive answers.

Issue c) has a more complex answer. The samples present many differences which are favourable to ES versus CS, even if, often, without a statistical relevance. Table 3 affirms that ES pupils show a greater confidence with FT questions since there was an improvement of performance from PT to FT (Table 4). In this sense the treatment had a good effect. We can assume ES children were in better position for connecting new and treatment knowledge (Mayer, 2002). But the competence performances (Table 4) do not support this statement, even if diminution is favourable to ES. Therefore we could conclude that there is a wide field of research to be investigated assessing our issue c).

NOTES

1. Work done in the sphere of Italian National Research Project Prin 2008PBBWNT at the Local Research Unit into Mathematics Education, Mathematics Department, Parma University, Italy.

2. We thank teachers Lucia Ferrarini (Vigatto - PR), Agnese Tomasini (Vicofertile - PR) for their participation to ES;
they presented activities with mirrors and paper folding in grade 1 and 2. We thank teacher Giordano Mancastroppa (‘Corazza’ of Parma) for CS. From PT to post-test (here named final test, for a distinct acronym, FT) the samples changed for occasional or definitive absence of some pupils. Our samples are reduced to the ES 38 pupils and the CS 33 children which took part to all PT and FT activities. The statistics are based on these reduced samples.

3. The PT presented three sheets (30 minutes each) which were administered in different days. The issues are freely inspired from literature: Shepherds from Marchetti et al. (2006), Pizza from Vighi (2010) and Pattern from I.Q. folklore. Notice that Vigatto schoolboys treated the original Vighi (2009) issues in the school year 2007/2008 (when they are 2nd graders). We assume that difference of questions and elapsed time made this previous experience irrelevant.

4. In case of $\chi^2$-test a datum is statistically significant if the test gives probability less than or equal to 1%, datum is statistically probable when the same value is less than or equal to 5% and greater than 1%. In case of statistics ‘$z$’, if $z > 1.96$ or $z < -1.96$ the probability of the equality for samples is lesser than 5% thence difference is statistically probable. If $z < -2.58$ or $z > 2.58$ this probability is less than 1%, thence difference is statistically significant. In the tables we single out with boldface font the relevance of a statistical test.

5. I.e. the rule which is explicated for the first four tiles is extended to the whole protocol (cf. Marchini & Vighi, 2011).

6. With Perim$_n$ (Area$_n$) we refer to the task of computing perimeter (area) of Shape$_n$.

7. Plane isometry is generally defined as a bijection of the plane in itself preserving distance of points. In the mathematical jargon, the adjective ‘local’ is not customary for isometry. With ‘local isometry’ we want to consider a bijection such as some part of the figure remains fixed and some other parts of the same figure are isometric. Thence a ‘local isometry’ could be globally an example of a non-isometric transformation. The Enclosure examples can explain this concept.

8. From a pure phenomenological point of view, we can use ‘trial’, instead of ‘confidence’. But we consider that ‘confidence’ resumes a positive behaviour towards a task and its gist, as a sort of container of trust, familiarity, self-reliance, poise, assurance, and so on.

9. The comparison of FT with PT is realized as follow. For each pupil the change of performance in confidence (in competence) is given by the sign of difference between the sum of results of FT task, and the corresponding PT issues, both normalized at 1, dividing by the number of tasks.

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Enclosure
Calculate perimeter and area of these shapes (measures are in centimetres). Then explain your solution.

Calculate the area of the following shape.