STATIC AND DYNAMIC APPROACH TO FORMING THE CONCEPT OF ROTATION

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Our research was based on arranging many different didactical situations that could be the source of intuitions for different geometrical transformations. In this case study, we deal with the concept of rotation. On the basis of different experiences, we were looking for the answer for a research question: how should we create a coherent picture of a geometrical transformation that functions as the one that enables us to understand the concept both statically and dynamically. The analysis of student’s behavior displayed number of common properties related both to static and dynamic understanding of that concept. Our work also reveals a fact that intuitions can contain elements which are inconsistent with the definition of rotation.

Key-words: isometries, geometrical intuitions, rotation.

INTRODUCTION

Theories that refer to Piaget’s depiction of internalization of actions state that mathematical concepts (even the simplest ones), theorems and language of mathematics are of operational nature.

According to Piaget, in case of logical – mathematical concepts, we encounter the interplay of operations, separated not from the perceived objects, but from the actions taken on them (Piaget & Inhelder, 1999). In Piaget’s view, transformation of reality is of fundamental significance and action is the tool of that transformation. In accordance with this idea, perception (vision) influences the formation of logical – mathematical thinking to a small degree; including geometrical thinking. To support such a standpoint, Aebli quotes Piaget’s views:

Investigating what activity is itself, we repeatedly verified its great importance contrary to the importance of an image. Geometrical view is, indeed, an active one as it mostly consists of potential actions, shortened schemata of effective actions or anticipatory schemata of future actions: in case of the lack of effective action, the view is inadequate (Aebli, 1982).

Uncritical implementing of Piaget’s views onto the field of geometry methodologies raises many objections worldwide (Clements, Battista, 1992; Clements at al., 1999, van Hiele, 1986). It is generally believed that the development of geometrical concepts is different from that of the arithmetical ones (Gray, Pinto, Pitta & Tall, 1999; Hejný, 1995, Vopěnka, 1989). The process of forming geometrical concepts has been the focus of a number of theories, of which van Hiele’s (1986) is the most popular. He describes the first level of understanding as “visual”, connected with non-verbal thinking. At this level the emphasis is
placed on the ability of recognizing shapes, which are perceived as a ‘whole’ and connected with visual prototypes. Not much is mentioned about the role of action, although a didactical conception of the theory suggests activities with objects (de Lange, 1987).

**Static Arrangement Figure To Figure**

If we accept the fact that the view is of significant importance on the first level of geometrical cognition, we also have to consider psychological provisions concerning cognition. Results of psychological research (Kaufman, 1979) confirm that in the process of grasping shapes pictorial designates are of great importance. In addition to that, dominance of the whole over the part is the regularity in perceiving shapes. The rules of structuring an image investigated in view of the information analysis system suggest that regular, symmetrical forms and shapes are the most easily recognized as one element can be predicted judging from the other (Grabowska & Budohoska, 1992). Regularities, groups creating some logical wholeness can be elements of a composition regulated through visual perception.

W. Demidow (1989) gives a broad account of the research conducted by physiologists concerning mechanisms governing shapes’ recognition. We can also find there information about invariant transformations conducted by our eyesight. For example, pictures of different sizes are invariant (unchangeable) to the organ of sight (the eyesight identifies them), the same happens while changing the position of an object - but only up to 15 degrees. The mirror image is not invariant even though children are born with such property of perception, as humans develop, an eye loses the invariance of mirror images.

These remarks have an essential meaning in geometrical environment, referred to as ‘patterns’. Creating bands or mosaics was unequivocally assessed by van Hiele as operating on the visual level that did not require internalization of actions. He refers to the structures of the first level as optical, structures of the appearance; they are manifested in recognizing regularities or certain wholeness. According to this theory all perceived regularities are classified as visual structures. The things that inspire children, propel them to action and which undergo control and are reflected upon are: rhythm, order and regularity. Such action seems to be in accordance with the original meaning of Greek ‘symmetros’ which stood for ‘harmonious’, ‘well-proportioned’.

The aforementioned sense of order tends to be verified visually by children. During the creation of geometrical compositions, the creative process is regulated by perception. Hence, propedeutics of geometrical figure to figure relation may reside in the sense of certain order, harmony - specific arrangement of surface or available fragment of space.

This leads us to the conclusion that in situations where balance, stemming from an appropriate arrangement of elements that constitute an image, is present,
there is no need to introduce movement. Children working in an environment of visual regularities do not recourse to the idea of movement, placing one object onto the other. This interpretation resembles the assessment of the mosaics that has been created by humans since the earliest days of their history. According to some historians of this discipline, mathematical relations can already be identified in the geometrical decorations of items created by the late ice-age man. In the book by Kordos (2005, p. 23) we read that:

It is worth paying attention to the richness of geometrical forms used in decorations. In particular, it is worth seeing that the ribbon ornaments from the Neolithic period all had 7 one-dimensional crystallographic groups on the surface. (...) However, we cannot be certain that some kind of geometrical reflection was followed.

Therefore, it seems that recognition of a specific figure to figure position is only a static image of this relation, not connected with moving of one object onto the other. On a certain level and in a certain context, it will be a rather general depiction, adjusted by the perception of certain regularity.

**Dynamic Understanding**

The understanding of relations between the figures as dynamic arrangement of space is placed, so to say, on the opposite pole. Acts of perception are important but they are not a sufficient source of geometrical cognition. Szemińska (1991, p.131) states that perception gives us only static images; through these, we can only catch some states, whereas by actions we can understand what causes them. It also guides us to possibilities of creating dynamic images.

The history of mathematics as a scientific discipline shows the importance of the transition from a static to a dynamic interpretation of geometrical objects (Kvasz, 2000). This can be seen in Greek mathematics, in which the traces of general reasoning were based on dynamic object transformations. A significant part of this geometry was based on constructions, which – in an indirect way – required the use of translations, rotations and mirror reflections. The overt description of symmetry as a transformation appeared rather late in mathematics – as it can be linked to the Erlangen Programme of F. Klein – but the dynamic approach itself is crucial for geometry. Geometrical reasoning requires mental transformation of objects.

In order to understand a geometrical symmetry as a transformation (translation, rotation or axis symmetry) it is necessary to conceive the specific movement that is transforming the initial figure into the final one. From a didactic point of view, it is important that such conception stems from mental reflection on the movement phenomenon. This is the idea of transformations as a function. If we want to trace the origins of this concept in the realm of physics, physical movement of an object will be suitable here. Nevertheless, such transformation happens in a given time and during making a movement and we can possibly trace the trajectory of an object. Everyday experience does not offer the
possibility of recording consecutive stages of the object’s movement. On the other hand, widely known Piaget’s results (Piaget & Inhelder, 1973) show that children (on the pre-operational level) have great difficulties in movement reproduction – they are not able to foresee a movement of an object in space. The process of acquiring such skills is lengthy and gradual (Szemińska, 1991). During manipulations, the child’s attention should be focused on action, not on the very result of action. It requires a different type of reflection than the one that accompanied his or her perception.

Such a problem is unnoticeable in school practice. Generally it is stated that creating patterns by children is a good starting point for their understanding of transformations. In some handbooks for teachers, there are suggestions to make exercises with changing the figure position like drawing patterns, mosaics where translation, rotation and mirror symmetry is used... Jones, Mooney, (2003), while analyzing school curricula in United Kingdom states, that the link between symmetry and the various transformations is not always made explicit. In the “Framework”, for instance, rotation appears to be considered solely as a transformation and the opportunity is missed to extend this to include rotational symmetry, even though the latter is specified in the statutory National Curriculum.

RESEARCH ORGANIZATION

Our research was based on arranging many different didactical situations that could be the source of intuitions for different geometrical transformations. Tiles were the basic tool for all situations. Patterns imprinted on tiles were different but they all had one general rule: on a piece of paper, one has to arrange something from tiles. Pupils from different age groups – from 5 year old children to gymnasium students took part in the research. This was a multistage research that started in 2002 and lasted until now. Organization of various stages was different. In some situations a work of a large group of students were analysed (more than thousand pupils), in the others – we observed lessons with only 20 students. Children worked at groups, during their regular activities. Some working sessions were videotapes. We observed and analyzed children’s behavior during their work and we analyzed their worksheets. On this basis, we estimated to what level classes that we propose can be treated as the basis for creating the picture of specific isometries by children.

In this case study, we will deal with the concept of rotation. We will present examples of results and commentaries connected with the following didactical situations:

- Creating a tiled floor by 4-6 year old children
- Creating a free tile composition by 10 – 13 year old children
- ‘Guided puzzle’ with a suggested subject area and musical background.
- ‘Domino’ task ( 12 year old children )
The first two situations concerned static situations, the following two were associated with movement.

Research questions were as following:

1. What intuitions connected with rotation will appear either in static situations or dynamic ones?

2. Which of those intuitions should we enhance and which should we lower during the further stages of mathematical education?

3. On the basis of different experiences, how should we create a coherent picture of a geometrical transformation that functions as the one that enables us to understand the concept both statically and dynamically?

**RESEARCH AND OBSERVATION RESULTS**

In children’s work, intuitions of rotation appeared in different forms. Each of them stressed a different rotation property that altogether exist in a mathematical understanding of the rotation on plane.

**Static situations**

A. Puzzles – filling the floor

a) Arrangement of one figure to another under a certain angle.

   Ania, 6 years old. In this work, the child tried to fill the plane with congruent figures arranged one to another under a certain angle. The size of the angle was dependent on the shape of the tool but it is clearly seen that the child was interested in a frequent change of the basic figure’s position.

b) Arrangement of figures around a certain centre. Figures in this arrangement do not have to change their positions in relation to privileged directions. They only ‘surround’ one chosen element.

   Julia, 6 years old. The process of gluing started from the central part of the paper sheet. Then, the girl tried to engird the circle with bells but she managed to do it only partially. In the following part, she focused on other, different regularities.
Milosz, 6 years old. A four-leaf clover was a central figure in his composition. The rest was submitted to stressing the central figure, surrounding it with contrasting elements and closing it within symmetrical frames.

c) ‘Along the edge’ arrangement, directed inwards. In certain places, especially at the corners of a sheet of paper, motives are created. They are directed to a common point of the inside – diagonal intersection

Boy, 7 years old. He started gluing from the frame. He managed to construct two elements in the upper parts in the way that they are symmetrical to each other. However, despite visible attempts to create corresponding elements, he could not repeat the same thing for the lower parts. In the central part, he only placed a symmetrical figure consisting of 4 tiles.

B. Puzzles – free activities

Older pupils, whose task in the initial stage was to arrange a free tile composition, worked in similar ways to those described above. Many of those works spontaneously realized the idea of symmetry – axis symmetry was the most frequent one. Going ‘beyond’ such arrangement did not happen often. In the observed group there were only two works where we can find traces of the idea of rotation.

Krzysiu, 13 years old. The pupil started his work from circles placed in the opposite corners of the paper (but the placement of dots in not correct). Then, he diagonally moved inwards. At the end, he glued elements at the corners. The placement of glued elements shows that point symmetry was present in his arrangement strategy.

Ala, 13 years old. Her work is an example of a perfect rotation, while the parts of the puzzle have an opposite orientation (the central part – clockwise orientation, circles in the corners – anticlockwise orientation). Rotation was present in the girl’s arrangement strategy from the beginning because it is impossible to see axis symmetry in her puzzle.
Both of these works can be classified to categories distinguished in 6 year old children’s works. First of these reflects the ‘along the edges’ arrangement idea, where the corresponding elements are in the opposite corners of a rectangular sheet of paper. Some deviations from the arrangement reflecting real rotations (elements marked with an oval) indicate, that the child, during her work, did not make any manipulations of a sheet of paper to check the arrangement of elements.

The second work is dynamic but compositionally close to the ‘around the centre’ arrangement. A closed, central composition arranged accordingly to a rotational movement by a closed, wavy line sticks out to the foreground. Rotation also organizes four smaller sets surrounding the centre, but the direction is different. It is clearly seen that movements are local. The whole composition is enclosed within a symmetrical area determined by a sheet of paper.

**Dynamic situations**

C. „Directional puzzles”

In another stage of the project with specially prepared music, the children created puzzles with a specific theme. Music functions in a natural way, speaks of a sort of transition and changes from moment A to moment B. In the assumptions, we referred to building dynamic associations with visual representations created by the child. Suggested theme was *merry-go-round*—the use of rotation.

The music and the topic actually inspired pupils to create compositions in which one could see relations connected with the idea of rotation. Placing several tiles required many full turns of single elements. The observation of the process proved very interesting. Some of the works stared from distinguishing the central element and then other elements were arranged around it — rotational arrangement. A two-dimensional pattern with rotational symmetry was formed. Maintaining only rotational symmetry was difficult, as it can be seen in the children’s works (fig. b,c).

Regardless of the external similarity of these creations to works created in the previously described stages, these puzzles tried to represent the idea of a specific movement. It results mainly from the way of organizing pupil’s work. Some children added tiles with regard to the rhythm of the music. There were also

![Diagram](image-url)
works where axial symmetry gradually transformed into rotations. Observation of the children’s work did not pose any doubts that they try to match the constructed arrangement with rotation—the pupil would draw an oval line with their finger, trying to see if the tiles go round one after another (fig.e). While doing this, the pupil would adjust the tiles and rearrange them in such a way that the dot pattern would represent rotation and not mirror symmetry.

D. „Dominoes”

The pupils (10-12 years old) have got a following task:

How many different „domino” blocks can be created by using two squared tiles with the motif presented in the picture below?

The first solutions were random. Pupils, sitting close to one another, could not recognize whether they have the same or different solutions. They also could not say whether these are all possible solutions. Moreover, they did not know if the arrangements in front of them are actually different from one another or the same.

Such situation was a good starting point for discussion and for more ordered way of looking for a solution.

An ‘unfailing’ strategy was proposed by a different pupil from the class. Below, we present his arrangement.

Analysis of this arrangement was a starting point for examining the position of one object towards another. This arrangement had a layout of a column: the boy started his arrangement from the first tile. A transition from one ‘domino block’ to the other happened through a conscious rotation of the second tile. Here, notions like ‘rotation by 90 degrees, 180 degrees, 270 degrees’ were appearing spontaneously. Although each domino block presented a relation of a rotation of two congruent figures, this relation was not the subject of research at this stage. For pupils, the way of constructing the whole series of dominos was very important, and this construction happened through rotating one of tiles.

**OBSERVATION CONCLUSIONS**

We conclude, that in the static recognition of rotation certain specific properties can be found:

✓ Rotation is understood locally.
✓ The center of rotation is an element which sticks out to the foreground. One of figures can serve the function of it.

There are elements which are inconsistent with the definition of rotation:
✓ During arrangement on a plane, there may be a lot of figures which are not congruent to one another.
✓ Figures that determine rotation themselves (around a given centre) are not rotated towards one another by a given angle. Their shift is rather parallel.

At the intuitional level of comprehending rotation, a *dynamic approach* can be characterized by the following:
✓ Movement representations are varied, strictly connected with a physical movement representative.
✓ They express only single definitions of rotation properties.
✓ A physical rotation of one figure results in the removal of the rotation center from the interest domain.

**SUMMARY**

We believe that regular ways of filling a sheet of paper with tiles can be treated as an intuition of geometrical transformations, even though initially they are not connected with the interiorization of movement. As far as quality is concerned, this knowledge is different than the mathematician’s knowledge, mainly because of the fact that it functions on totally different rules. If we want certain relations to be clear for children, we need to introduce a ‘rich structure’ in which not only two figures (e.g. polygons) remain in a certain relation to each other but a certain fragment of space is organized accordingly to this relation. Introduction to the understanding of geometrical relations that function in mathematics as a science is created through the feeling of regularity on a statically organized plane. Here, a child can arrange, organize a sheet of paper and ideas rise at the moment of reflection on what he or she sees.

Exterior effects of pupil’s works who create representations for a rotation, created both in a static and dynamic environment do not vary that much. It does not mean though, that these approaches can be identified with each other. In both approaches, the organization of pupil’s work which pointed at different pictures associated with the performed activity was different. However, since these works (as a final effect) look alike, they give a possibility of building an integrated static-dynamic picture.

Presented didactical examples are not just ‘clear’ models of the mathematical notion of rotating by any angle. It is consistent with our understanding of the constructivistic approach towards the creation of mathematical concepts. A pupil should function in such a rich learning environment that, through gaining various experiences and reflecting upon them, he would be able to create his own understanding of isometric transformations.
In spite of this, the relation between visual recognition of geometrical objects and actions that can lead to the creation of dynamic images of those objects needs further investigation.

REFERENCES

Jones, K., Mooney, C. (2003), Making Space for Geometry in Primary Mathematics.