RELATING GRAPH SEMIOTIC COMPLEXITY TO GRAPH COMPREHENSION IN STATISTICAL GRAPHS PRODUCED BY PROSPECTIVE TEACHERS

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In this paper the graphs produced by 207 prospective primary school teachers in an open semi-structured statistical project where they had to compare three pairs of statistical variables are analysed. The graphs are classified according their semiotic complexity, and the teachers' levels of comprehension in Curcio's (1989) categorization. Most participants produced graphs with sufficient semiotic complexity to solve the task proposed; however, the correct conclusion was only reached by a minority of prospective teachers who were able to read the data produced at the "reading behind the data" level. When relating semiotic complexity of graphs to the reading level, teachers producing graphs at the highest semiotic level also reached the highest combined percentage of "reading beyond data" and "reading between data" levels.

Keywords: statistical graphs, semiotic complexity, graph interpretation and comprehension.

INTRODUCTION

Graphical language is an important part of statistical literacy (Watson, 2006). It is also a tool for transnumeration, a basic component in statistical reasoning consisting of "changing representations to engender understanding" (Wild & Pfannkuch, 1999, p. 227). In this work we complement our previous research on Spanish prospective primary school teachers' graphical competence (Batanero, Arteaga & Ruiz, 2010) with the aim to relate the semiotic complexity of graphs produced by prospective primary school teachers with the graph comprehension levels defined by Curcio (1989).

Understanding Statistical Graphs

Previous research suggest that competence related to statistical graphs is not reached in compulsory education, since students make errors in establishing the graph scales (Li & Shen, 1992) or in building specific graphs (Pereira Mendoza & Mellor, 1990; Lee & Meletiou, 2003; Bakker, Biehler & Konold, 2004). Several authors investigated levels in graph understanding. For this particular research we are using Curcio's categorisation (1989), that consists of the following levels: (a) *Reading the data,* is the level of a student who is only able to answer explicit questions for which the obvious answer is right there in the graph; (b) *Reading between the data* involves interpolating and finding relationships in the data presented in a graph. This includes making comparisons as well as applying operations to data; (c) *Reading beyond the data* involves extrapolating, predicting, or inferring from the representation to answer questions related to tendencies in the data or extrapolation from the data. In this research we are only considering these three levels, which do not imply tje need to look critically at the data. This would be a new level *looking behind the data,* according to Friel, Bright, and Curcio (2001).

Graphical Competence in Prospective Teachers

Few studies have focused on teachers' knowledge and conceptions about statistical graphs and most of these studies are related to prospective teachers (González, Espinel, & Ainley, in preparation). Results from this research highlight the scarce graphical competence in prospective teachers. For example, in a study conducted with 29 prospective primary teachers in Spain, Bruno and Espinel (2009) found frequent errors when building histograms or frequency polygons. In another study with 190 prospective primary school teachers, Espinel, Bruno, and Plasencia (2008) observed lack of coherence between the graphs built by participants and their evaluation of tasks carried out by fictitious future students. Monteiro and Ainley (2007) suggested that many Brazilian prospective teachers in a sample of 218 teachers did not possess enough mathematical knowledge to read graphs taken from daily press. In Burgess's (2002) study some teachers made graphs in their reports but were unable to integrate the knowledge they could get from the graphs with the problem context.

Batanero, Arteaga, and Ruiz (2010) analysed the graphs produced by 93 prospective primary school teachers in an open semi-structured statistical project where they had to compare two statistical variables. They defined a *semiotic complexity level* in these graphs and analysed the teachers' errors in selecting and building the graphs, as well as their capacity to reach a conclusion on the research question. Although about two thirds of the participants produced a graph with enough semiotic complexity to get an adequate conclusion, half the graphs were either inadequate to the problem or incorrect. Only one third of participants were able to get a conclusion in relation to the research question. In this paper we increase the sample size and the number of variables to be compared in the same semi-structured statistical project in order to relate the graphs semiotic complexity with the prospective teachers' level in reading the graphs.

Semiotic Complexity in Statistical Graphs

In mathematical work we usually take some objects (e.g. a symbol or a word) to represent other abstract objects (e.g. the concept of average). In these situations, and according to Eco "there is a *semiotic function* when an expression and a content are in correlation" (Eco 1979, p. 83). Such a correlation is conventionally established, though this does not imply arbitrariness, but it depends on a cultural link. Font, Godino, and D'Amore (2007) suggested that all the different types of objects that intervene in mathematical practices: *problems, actions, concepts, language, properties and arguments* could be used as either expression or content in a semiotic function. In our study we proposed an open semi-structured project to prospective teachers. To address the project, the participants had to solve some mathematical *problems* (e.g., comparing three different pairs of distributions) and perform some mathematical practices to solve these problems. The focus in our research is the statistical graphs produced by the teachers and the mathematical practices linked to the different types of graphs. When teachers produce a graph they need to perform a series of *actions* (such as deciding the particular type of graph or, fixing the scale), they implicitly used some *concepts* (such as variable, value, frequency, range) and *properties* (e.g. proportionality between frequencies and length of bars in the bar graph) that vary in different graphs. Consequently the semiotic functions implicit in building each graph also varies. We therefore should not consider the different graphs as equivalent representations of a same mathematical concept (the data distribution) but as different configurations of interrelated mathematical objects that interact with that distribution. Taking into account these ideas, Batanero, Arteaga, and Ruiz (2010) defined different levels in graphs semiotic complexity, as follow (see examples in Figure 1).

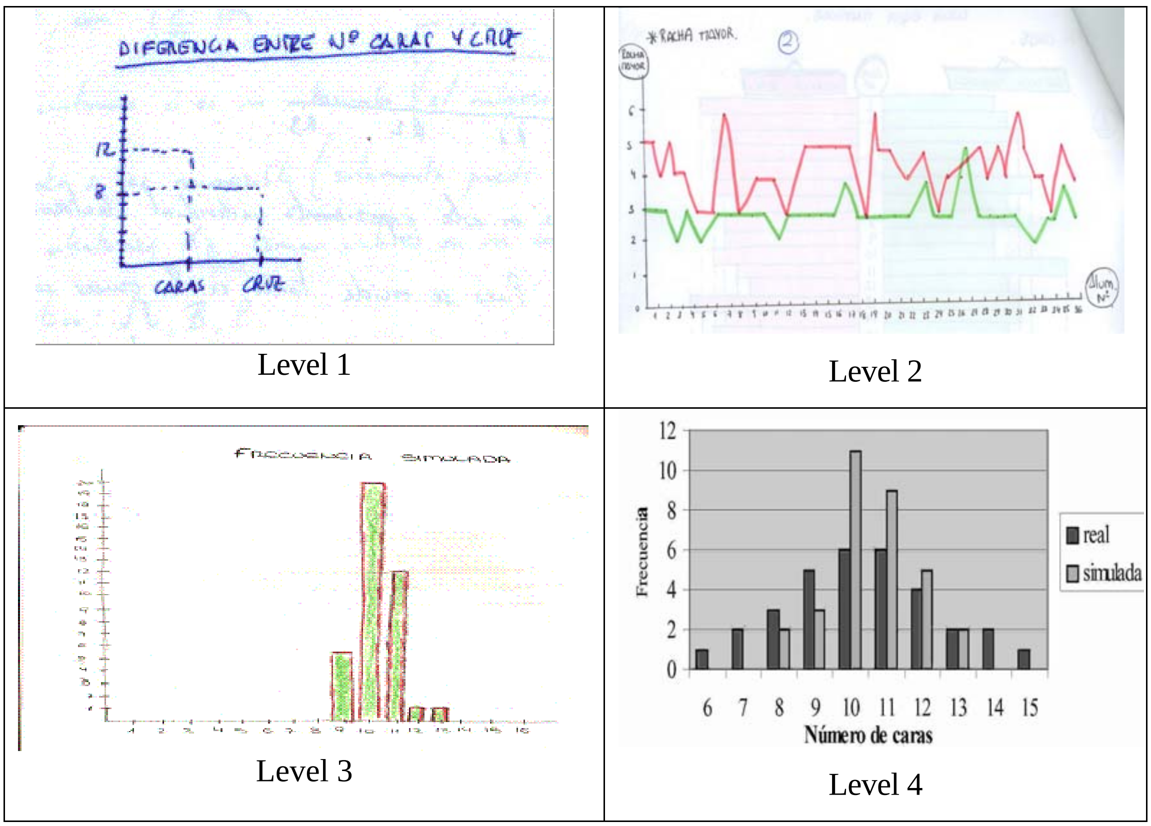


Figure 1. Examples of graphs at different semiotic complexity level

L1. Representing only individual results. When given a set of data, some students do not complete the graph for the whole data set; instead they only represent isolated data values. When the data are collected in the classroom they only represent their own data, without considering their classmates' data, for example, they represented the number of heads in his /her individual experiment. These students do not use the idea of statistical variable or distribution when producing their graphs.

L2. Representing all the individual values for one or several variables, without forming the distribution. Some students do not form the frequency distribution of the variables, when they are given a data set. Instead, they produce a graph where data are represented one by one, without an attempt to order the data or to combine identical values. Consequently these students neither compute the frequency of the different values nor explicitly use the idea of distribution.

L3. Producing separate graphs for each distribution. When comparing a pair of distributions, some students use the idea of frequency and distribution but produce a separate graph for each variable to be compared. Often, these students use either different scales in both graphs or different graphs for the two distributions, which makes the comparison hard.

L4. Producing a joint graph for both distributions. This level corresponds to students that form the distributions for the two variables to be compared and represent them in a joint graph, which facilitated the comparison. These graphs are the most complex, since they represent two different variables in the same frame.

THE STUDY

Participants in the sample were 207 pre-service teachers in Spain, in total 6 different groups (35-40 pre-service teachers by group). All of them were following the same mathematics education course and studied descriptive statistics at a secondary school level, and in the previous academic year (for 20 hours), where they had worked with another statistical project. In this paper we analyse the graphs produced by these teachers when working in a semi-structured statistical project in which participants were asked to perform a random experiment, collect data, compare three pairs of distributions and come to a conclusion about the group's intuition of randomness, basing their conclusion on the analysis of the data. The sequence of activities in the project was as follows:

1. *Presenting the problem, and experiment.* We proposed that the prospective teachers carry out an experiment to decide whether they had good intuitions on randomness or not. The experiment had two parts. In the first part (simulated sequence) each participant wrote down apparent random results of flipping a coin 20 times (without really throwing the coin, just inventing the results) in such a way that other people would think the coin was flipped at random. In the second part (real sequence) each participant flipped a fair coin 20 times and wrote down the results.
2. *Collecting data and instructions.* Each prospective teacher performed both experiments. After the lecturer started a discussion about how the simulated and real sequences for the whole group could be compared, some students suggested to

collect data from the number of heads or number of runs. Finally the lecturer suggested comparing the following statistical variables: number of heads, number of runs and longest run. Each prospective teacher provided his/her results in each of the variables that were recorded on a recording sheet. At the end of the session the prospective teachers were given a printed copy of the data set for the whole group of students. They were asked to produce a written report including their statistical analyses and their conclusions. There was no restriction in the report length; teachers were given freedom to use any statistical method or graph they wished and were allowed to use computers. They were given a week to complete the reports (for more details of the project, see Godino, Batanero, Roa & Wilhelmi, 2008).

RESULTS AND DISCUSSION

Once the prospective service teachers' written reports were collected, we made a qualitative analysis of these reports. From a total of 207 students 181 (87,4%), 146 (70,5%) and 128 (61,8%) produced some graphs when analysing the number of heads, number of runs and longest run, even if the instructions given to the students did not explicitly ask them to construct a graph. These high percentages suggest that prospective teachers felt the need of building a graph to reach, through a transnumeration process (Wild & Pfannkuch, 1999) some information that was not available in the raw data. In Table 1 we present the results. These data show the relative difficulty of the variables to be analysed, as the number of heads was more familiar to the teachers than the runs.

Semiotic complexity	N. of heads	N. of runs	Longest run
L1. Representing only individual data	6 (3.3)	6 (4.1)	3 (2.3)
L2. Representing the data list	40 (22.1)	23 (15.7)	21 (16.4)
L3. Producing separate graphs for each distribution	91 (50.3)	77 (52.7)	67 (52.3)
L4. Producing a joint graph to compare both distributions	44 (24.3)	40 (27.4)	37 (28.9)
Number of participants producing graphs	181	146	128

Table 1. Frequency (percentage) of participants producing graphs in each semiotic level and pair of variables

Few participant produced level L1 graphs, that is, only analysed their own data and less than 25% represented the data list in the same order given in the data sheet without making an attempt to summarise the data, producing the variables distributions. Consequently the concept of distribution seemed natural for the majority of students who produced a graph, since about 75% of them built a distribution to compare each pair of variables, although the instructions did not explicitly require this. Results agree with those reported by Batanero, Arteaga, and Ruiz (2010).

Graph comprehension

In table 2 we classify participants according to Curcio's (1989) categorization of graph comprehension, in the following way:

R0. *Do not read the graph or incorrect reading*: About 30% of the teachers in each pair of variables only produced and presented the graph in their report with no attempt to read the graph, and reached no conclusion about the problem posed. In addition, after producing the graph, between 11% to 14% of the teachers in each pair of variables failed when reading the information. Some of these failures were produced by errors in the graphs that reproduced those described in Bruno and Espinel (2009) or incorrect choice of the type of graph that was inadequate for the information represented in the graph. Other failures in reading the graph were due to incorrect understanding of a concept; for example confusing frequencies and values of the variable or misinterpreting the meaning of the standard deviation.

R1. *Reading data:* Between 22- 25% of the participants made a correct literal reading of graphs labels, scales and specific information on the graph in each pair of variables. However they only considered superficial features of the graph. For example, they compared isolated values of the two variables, provided the frequency for a given value or made a general comment about the shape of the graph with no consideration given to tendencies or variability in the data.

Graph comprehension level (Curcio)	N. of heads	N. of runs	Longest run
R0. Do not read the graph	51 (28.2)	45(30.9)	42(32.8)
R0. Incorrect reading	21(11.6)	17(11.6)	18(14.1)
R1. Reading data	41(22.6)	34(23.3)	32(25)
R2. Reading between data	44(24.3)	32(21.9)	21(16.4)
R3. Reading beyond data	24(13.3)	18(12.3)	15(11.7)
Number of students producing graphs	181	146	128

Table 2. Frequency (percent) of participants producing graphs according to comprehension level

R2. *Reading between data*: Teachers classified in this level were able to make comparisons and look for relationships in the data. They either compared averages (means, medians or modes) alone in both distributions (with no consideration of variation in the data) or else compared spread (without comparing averages).

R3. *Reading beyond data:* Making inferences and drawing conclusions from the graph:

Participants at this level were able to compare both spread and average in the distribution and conclude about the differences taking into account both data features.

Less than 25% of tje prospective teachers who built graphs reached the Curcio's (1989) intermediate level (reading between the data) and only 13% reached the upper level (Table 2). Notice that the percentage of students building graphs without interpreting them is high, which agree with Burgess (2002). The difficulty of reading the data increased for variables related to runs that were less familiar to participants.

In Figure 2 we take into account only the prospective teachers who interpreted the graphs they built themselves, classifying their representations according to the graph semiotic complexity level and reading comprehension level for each pair of variables and for all the graphs combined. The Chi-square test to check independence of reading levels and semiotic complexity levels for all the graphs combined was statistically significant (Chi=40.4, dg=9, p<0.0001) and therefore we can accept that these two types of levels are related.

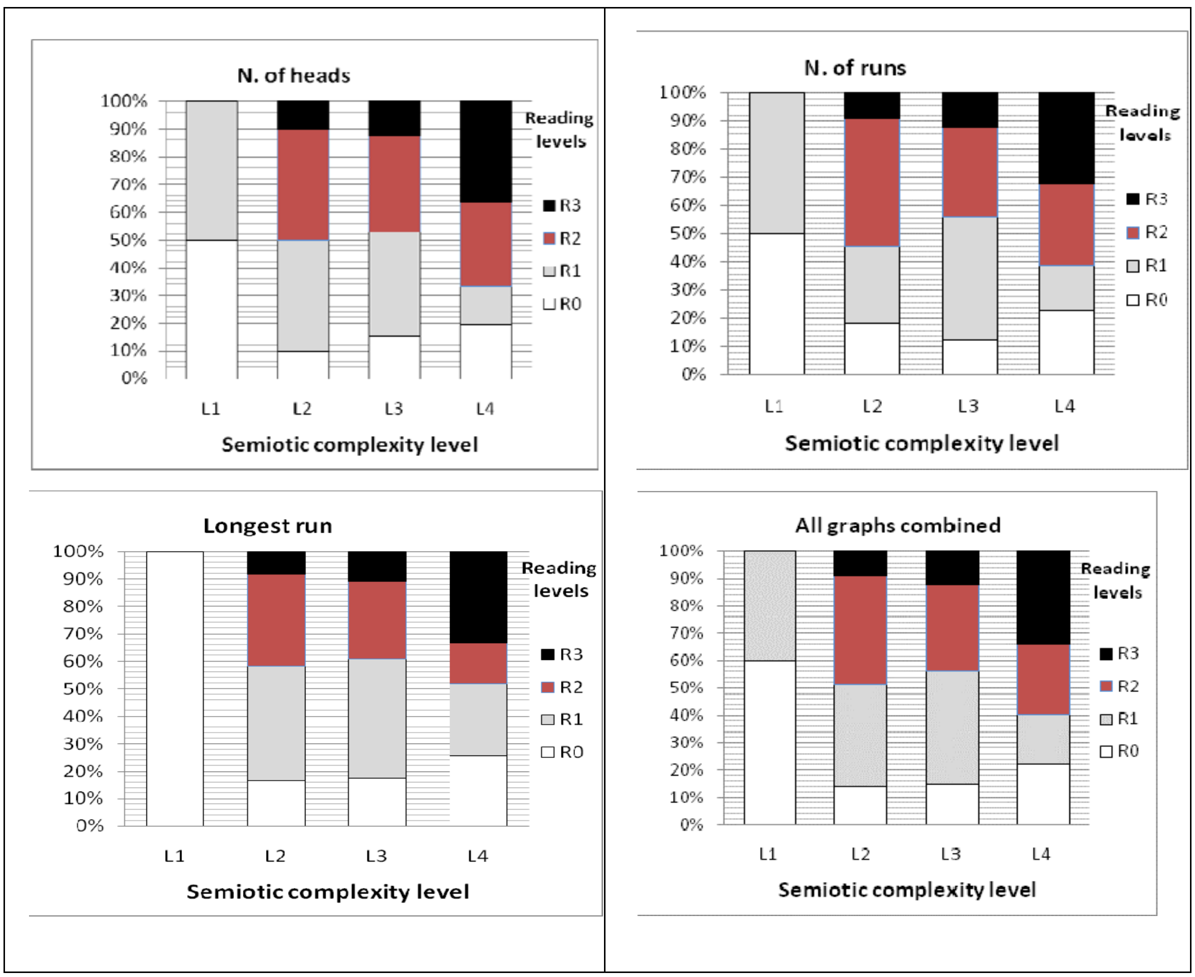
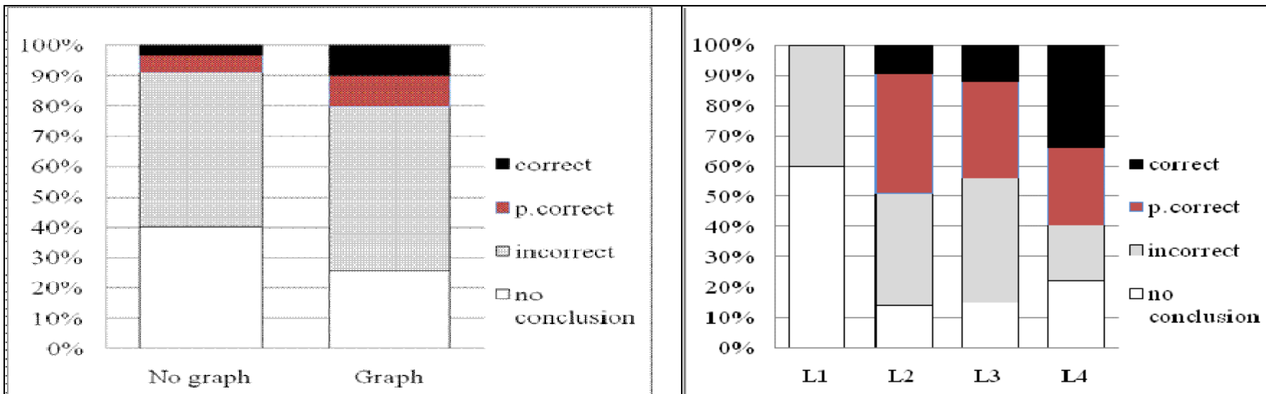


Figure 2. Reading levels by semiotic complexity in the graph

Prospective teachers producing semiotic level L1 graphs either made an incorrect reading or only reached the literal “reading data” level. The percent of incorrect reading dramatically decreased in the remaining levels; however this percent is higher in level L4 than in levels L2 or L3. This is probably because more complex graphs were harder to be interpreted correctly by participants. Although there is no clear tendency as regards literal “reading the data” level, reading the data only is not productive to reach a conclusion on the problem posed. “Reading between data” level is more frequent in level L2 graphs, because in these graphs the data variability is very easily perceived (as compared with levels L1 or L3 graphs). The highest percentage of “reading beyond data” level, when teachers are able to analyse both the tendency and spread in the data and reach a conclusion, as well the combined percentage of “reading between data” and “reading beyond data” levels were reached in semiotic level L4 graphs because in level L4 graphs students can perceive spread and tendencies more easily. Therefore level L4 graphs provide more opportunity for students to get at least a partly correct reading. Consequently, it is important that the teacher’s educator try to promote higher reading levels when possible.



a. Conclusions according production of graph

b. Conclusions according graph semiotic level

Figure 3. Conclusion according to (a) production of graph; (b) graph semiotic level

In the project proposed the students should reach a conclusion regarding the group intuition on randomness. Only 8.9% of those prospective teachers who produced no graph got a correct or partly correct conclusion. This percentage increased to 20.2% in those teachers who produced graphs as part of their analyses (10,2% and 10 %); the differences are significant in the Chi-square test (Chi=18,72, d.g.=3; p=0.007). Therefore, building a graph helped the teachers in their conclusions (Figure 3.a). The percentage of correct conclusions increased to about 30% in teachers producing level L4 graphs (Figure 3.b), because, at these levels the teachers read the graph at a higher reading level and in these graphs the perception of both tendencies and spread is easier. The percentage of partly correct conclusions was higher at level L2 graphs, because of easy perception of variability in these graphs. The Chi-square test to check independence of type of conclusions and the semiotic complexity levels for all the graphs combined was statistically significant (Chi=40,45; d.g.= 9; p= 0,0000) and therefore we can accept that these two variables are related.

IMPLICATIONS FOR TEACHER EDUCATION

In agreement with Bruno and Espinel (2009) and Monteiro and Ainley (2007) our research suggests that building and interpreting graphs is a complex activity for prospective school teachers. We agree with these authors in the relevance of improving the prospective teachers' levels of competences in both building and interpreting graphs, so that they can later transmit these competences to their own students. Many participants in the study limited their analysis to producing graphs with no attempt to get a conclusion about the problem posed. Consequently, prospective teachers need more training in working with statistical projects, since working with projects is today recommended in the teaching of statistics from primary school level.

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