UNRAVELING MATHEMATICAL GIFTEDNESS

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This study purports to develop a multiple criteria identification process for mathematically gifted students, in an effort to clarify the construct of mathematical giftedness. The study was conducted among 359 4th, 5th and 6th grade elementary school students in Cyprus, using four instruments measuring mathematical ability, mathematical creativity, self-perceptions of mathematical behaviour and fluid intelligence. The results revealed that mathematical giftedness can be described in terms of mathematical ability and mathematical creativity. Moreover, the analysis illustrated that although self-perceptions and fluid intelligence do not consist mathematical giftedness, they could predict it. Implications for researchers and teachers are discussed.

Keywords: giftedness, mathematical ability, creativity, self-perceptions, intelligence.

INTRODUCTION

To meet the challenges of the new millennium, the field of giftedness research has to expand its conceptions with revised models and approaches (Ziegler, 2009). Despite the number of publications that introduced several conceptions of giftedness, only a small number of them have been empirically examined (Stoeger, 2009). Thus, the need for more empirical studies in the future is evident.

The field-dependent character of giftedness was pointed out by Csikszentmihalyi (2000). However, prior research in the field of giftedness focused on the examination of general giftedness rather than domain-specific giftedness. As a result, there is limited focus on theoretical models of mathematical giftedness as well as specially designed procedures and instruments for students’ identification.

This study attempts to complement the lack of empirical studies in giftedness and the lack of studies, models and identification processes focused on mathematical giftedness. Specifically, the present study aims to investigate the construct of mathematical giftedness in students aged 10-12 years old (4th-6th grades) and to develop an identification process based on a multiple criteria approach.

THEORETICAL FRAMEWORK

Conceptualizations of giftedness

Although IQ was for decades considered as the only and predominant index of giftedness, a major shift was later noted in the research field. Namely, environmental
influences were acknowledged resulting to the decrease of the influence of intelligence (Hartas, Lindsay & Muijs, 2008). At the same end, there were studies which have questioned the validity and liability to cultural and social bias of standardized measures such as IQ (Black, 2001). Despite the criticism, Silverman (2009) claims that instruments with the richest loadings on general intelligence, such as the Wechsler scales, are the most useful for identification for giftedness.

Turning away from intelligence as an indicator of giftedness, among the conceptualizations of giftedness that have been proposed over the years, a widely accepted definition was proposed by Renzulli (1978), emphasizing above-average ability and creativity as characteristics of gifted individuals. In the area of mathematics, the relationship between mathematical giftedness and creativity has also been documented by a number of researchers (e.g. Sriraman, 2005).

Identification of giftedness

Due to the lack of conceptual clarity as to the nature of giftedness, identification processes have varied widely. Identifying gifted individuals raises important issues regarding the types of evidence of giftedness and the validity of assessment processes, since gifted children will be provided with opportunities not accessible to others.

For gifted children to be identified, researchers should decide upon their discriminating characteristics and assess their ability in the specific domain, in our case mathematics. In addition to above average mathematical ability and mathematical creativity, researchers suggested that gifted students can also be identified by examining students’ learning pace, depth of understanding and interests (Maker, 1982). According to other researchers, information about certain personality characteristics of students (e.g., persistence, perseverance, resilience), motivation, and interests (Hartas, Lindsay & Muijs, 2008) should be collected. Equally prevalent is the desire to understand students’ perceptions with respect to these characteristics and behaviours.

Particularly in the case of mathematically gifted students, researchers should pay particular attention to mathematical abilities of highly able students and characteristics related to mathematical reasoning. For example, Krutetskii’s work (1976) revealed a number of characteristics and abilities that mathematically able children possess: ability for logical thought with respect to quantitative and spatial relationships, number and letter symbols; the ability for rapid and broad generalization of mathematical relations and operations, flexibility of mental processes and mathematical memory. Moreover, a number of characteristics of mathematical giftedness have been proposed by several researchers (Benbow & Minor, 1990; Feldhusen, Hoover, & Sayler, 1991; House, 1987; NCTM, 2000; Olszewski-Kubiliyus, Kulieke, Shaw, Wilhus, & Krasney, 1990; Sriraman, 2005; Stanley, 1993; Wieczerkowski & Prado, 1993), such as high spatial ability, the ability to develop unique relations, producing original, insightful solutions/methods for
solutions or formulating imaginative questions and the ability to organise data in such a way to consider patterns or relationships.

Following, this wide variety of characteristics of gifted individuals calls upon the use of a multiple-criteria approach during their identification, employing a combination of valid and reliable tools and multiple sources of evidence (Hoeflinger, 1998). Among other instruments, tests, self-report questionnaires, teacher rating scales, checklists and inventories have been introduced as measures of the identification of giftedness. To sum up, both evaluation of academic performance and cognitive abilities are used (Naglieri & Ford, 2003), despite their conceptual differences in combination with evidence of students’ perceptions.

 PURPOSE OF THE STUDY

Having in mind the abovementioned considerations, the purpose of this article is twofold. Firstly, the study attempts to investigate the construct of mathematical giftedness which comprises of mathematical ability and mathematical creativity. To clarify this concept, the relationship between students’ self-perceptions, fluid intelligence and mathematical giftedness is investigated. Secondly, the study purports to develop a valid and reliable identification process for identifying mathematically gifted students based on multiple measures. These measures assess mathematical ability, mathematical creativity, students’ self-perceptions with respect to mathematical behaviour and fluid intelligence.

To fulfil the purpose of the study, a theoretical model was a-priori created (see Figure 1). In this model, we hypothesized that mathematical giftedness consists of mathematical ability and creativity. Furthermore, we assumed that self-perceptions regarding students’ behavioural characteristics in mathematics and fluid intelligence would contribute to the prediction of students’ mathematical giftedness.

Figure 1: The proposed model.
METHODOLOGY

Sample and instruments
To fulfil the aims of the study, four instruments were administered to 359 students of age 9-12 years old; the mathematical abilities instrument, the mathematical creativity instrument, the self-report questionnaire and the fluid intelligence instrument. The mathematical instrument comprised of 29 mathematical items measuring spatial, quantitative, qualitative, causal and inductive/deductive abilities. The creativity instrument included five open-ended mathematical tasks. The self-report questionnaire consisted of 20 statements describing behaviours with special focus on mathematics. Students responded on a 5-point Likert scale regarding the frequency of each behaviour observed. To measure fluid intelligence, we used the subtest Matrix Reasoning Scale from the Wechsler Abbreviated Scale of Intelligence (WASI) (Wechsler, 1999). The WASI Matrix Reasoning Scale provides a measure of nonverbal fluid abilities using 32 tasks for students of 9 to 11 years old and 35 tasks for students older than 11 years. All instruments were group administered in electronic form except from the WASI Matrix Reasoning Scale which was completed individually, in a hard-copy form.

Data analysis
For the analysis of the data confirmatory analysis was employed using the statistical package MPLUS. In this study, confirmatory factor analysis (CFA) was used to investigate whether the proposed model for the identification of mathematically gifted students fits our data. In order to evaluate model fit, three fit indices were computed: The chi-square to its degree of freedom ratio ($\chi^2/df$), the comparative fit index (CFI), and the root mean-square error of approximation (RMSEA) (Marcoulides & Schumacker, 1996). For the model to be confirmed, the values for CFI should be higher than 0.9, the observed values of $\chi^2/df$ should be less than 2 and the RMSEA values should be close to zero.

RESULTS
For the construct validity of the model to be evaluated, a confirmatory factor analysis (CFA) was employed. CFA showed that all tasks and statements of the four instruments loaded adequately (i.e., they were statistically significant, because the z values were greater than 1.96) on each factor (see Figure 2). Figure 2, presents the structural equation model with the latent and observed variables and their indicators. CFA also showed that the observed (students’ responses to each task and statement) and theoretical factor structures (the components of the theoretical model) matched the data set of the present study and determined the “goodness of fit” of the factor model (CFI=0.923, $\chi^2=566.627$, df=366, $\chi^2/df= 1.626$, RMSEA=0.039).
Figure 2: The structure of the proposed model.
Therefore, the analysis suggested a model representing distinct components that should be considered during the identification of mathematical giftedness. Thus, (a) spatial abilities, (b) quantitative abilities, (c) qualitative abilities, (d) verbal abilities and (e) causal abilities constitute mathematical abilities while, fluency, flexibility and originality comprise mathematical creativity. Both mathematical abilities and mathematical creativity constitute mathematical giftedness. In addition, the analysis revealed that fluid intelligence and self-perceptions of mathematical behaviour could predict mathematical giftedness. Moreover, with regard to students’ self-perceptions about mathematical characteristics the data illustrated that they can be organised across five dimensions and should be considered during the identification of mathematically gifted students. In particular, students’ descriptions in regard to their (a) learning characteristics, (b) interests/curiosity, (c) creativity, (d) social-emotional characteristics and (e) mathematical reasoning are important for the identification of mathematically gifted students.

Specifically, the analysis revealed that the five types of mathematical abilities measured by the mathematical instrument constitute one general factor, that of mathematical ability (F1). In particular, the statistically significant loadings of spatial abilities (r=.332, p<.05), quantitative abilities (r=.671, p<.05), qualitative abilities (r=.632, p<.05), inductive/deductive abilities (r=.717, p<.05), and causal abilities (r=.455, p<.05) verify that these abilities constitute general mathematical ability. The data suggest that for this age group the quantitative, qualitative and inductive/deductive abilities contribute more than the causal and spatial abilities to mathematical abilities. Likewise, the loadings of fluency (r=.836, p<.05), flexibility (r=.925, p<.05) and originality (r=.790, p<.05) suggest that these three first order factors constitute the second order factor of mathematical creativity (F2). Furthermore, one general factor, that of mathematical giftedness (F100), was generated from mathematical ability (r=.915, p<.05) and mathematical creativity (r=.668, p<.05) as shown by their statistically significant loadings.

In addition, students’ self-perceptions of their characteristics with regard to mathematics comprise of five factors (F4-F8) with statistically significant loadings; learning characteristics (r=.786, p<.05), interests/curiosity (r=.837, p<.05), creativity (r=.970, p<.05), social-emotional characteristics (r=.999, p<.05) mathematical reasoning (r=.838, p<.05). The data suggest that according to students’ responses for this age group, characteristics describing social-emotional characteristics and characteristics describing creative behaviours contribute more than learning characteristics, interests/curiosity and mathematical reasoning to their self-perceptions.

The structure of the proposed model also addresses that students’ perceptions and fluid intelligence are able to significantly predict students’ mathematical giftedness (r=.216, p<.05 and r=.599, p<.05 respectively).
DISCUSSION

Given the controversy prevailing in the field of giftedness, reflected both in the variety of concepts and identification processes proposed, the identification of mathematically gifted students is considered to be extremely challenging (Hoeflinger, 1998). Hence, this study comes to complement the lack of empirical studies in giftedness as well as the lack of studies, models and identification processes focused on mathematical giftedness. Firstly, the study attempts to investigate the construct of mathematical giftedness. Namely, it was expected that the relationship between students’ self-perceptions, fluid intelligence and mathematical giftedness, composed by mathematical ability and creativity, would be clarified. Secondly, the study aims to develop a valid and reliable identification process based on multiple criteria and instruments. This process is focused on the identification of mathematically gifted students at the upper grades of the elementary school.

To fulfil the purpose of the study, a theoretical model was conceived and it was later empirically tested and confirmed. With respect to the first objective, data analysis revealed that mathematical ability can be defined in terms of five abilities; spatial conception (spatial ability), number relationships (quantitative ability), the ability of analogical thought (qualitative ability), experimentation skills (causal ability) and logical reasoning (inductive/deductive abilities), confirming the abilities suggested by Krutetskii (1976) as characteristics of mathematically able children. Mathematical creativity can also be described in terms of fluency, flexibility and originality, as proposed by Torrance (1974). Moreover, the findings showed that mathematical ability and creativity constitute a more general factor, that of mathematical giftedness.

The model extended the literature to include two specific measures that may predict mathematical giftedness, complementing the other two measures, the mathematical ability and the mathematical creativity instruments. Specifically, it was shown that although fluid intelligence and self-perceptions of mathematical behaviour are not components of mathematical giftedness, they could predict it. The finding for the predictive power of self-perceptions about mathematics is in accord with other researchers who claim that self-efficacy of gifted students contribute to the prediction of math performance (Pajares, 1996). Moreover, data analysis revealed that students’ responses to the self-report questionnaire can be organized across five distinct factors; (a) learning characteristics, (b) interests/curiosity, (c) creativity, (d) social-emotional characteristics and (e) mathematical reasoning. These characteristics have been mentioned by researchers listing the traits of gifted students (e.g. Hartas, Lindsay & Muijs, 2008; Maker, 1982).

The findings imply that the identification process of mathematically gifted students should include multiple measures in order to capture the variety of characteristics that these students present. Since we aim to identify mathematical and not general giftedness, instruments measuring mathematical ability and mathematical creativity are fundamental. Furthermore, the use of self-report questionnaires such as the one
reported in this study and a measure of fluid intelligence could be used as supplementary means to collect information about mathematical giftedness, since they have been found to predict it. In particular, the self-report questionnaire addresses a variety of characteristics focusing in mathematics. Therefore, it may complement the lack of domain-specific identification instruments for gifted students, in this area. At the same time, the fluid intelligence instrument sheds light on the relationship between intelligence and giftedness.

From the theoretical point, the model may inform the controversial concepts of intelligence, giftedness and domain-specific abilities. This model suggests valid evaluation of mathematical giftedness could be made both with processes where general and domain-specific abilities and processes are measured. In practice, the model is expected to facilitate the identification process of mathematically gifted students and to afterwards promote the nourishment of students’ mathematical talent. Namely, teachers could identify gifted students through the identification instruments which encompass the components of the theoretical model proposed. Certainly, further research is required in order to introduce and examine additional instruments that might contribute to the identification of mathematical giftedness.

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REFERENCES


